Heat Transfer

Heat Exchangers - Effectiveness-NTU Method

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Difficulties with LMTD Method

From the design equation:

$$Q = UA\Delta T_{\text{lm}}$$

From energy balance:

$$Q = \dot{m}_c C_{Pc}(T_{co} - T_{ci}) = \dot{m}_h C_{Ph}(T_{hi} - T_{ho})$$

The LMTD method of heat exchanger design is difficult to use if we want to predict the performance of a heat exchanger.

Here we would know: $\dot{m}_c, \dot{m}_h, T_{ci}, T_{hi}, U$, and A

However, we would not know: T_{co} or T_{ho}

Hence, we cannot find: Q or ΔT_{lm}

Difficulties with LMTD Method (contd..)

To solve the above problem with the usual LMTD method:

1. We could guess a value for T_{ho} or T_{co}, find Q from a heat balance, and then find Q from $UA\Delta T_{\mathrm{lm}}$.
2. Using this value of Q, find new values of T_{ho} and T_{co}.
3. We would need to progressively alter our guess until the first and second step values of T were equal.

This iterative method can readily be done by computer, but a direct method can also be used. This direct method is known as the Effectiveness-NTU method (or ε-NTU method)

Effectiveness - NTU Method

In order to use this method we need three new definitions:

1. Thermal Capacity Ratio (C):
   The thermal capacity of a fluid stream is the quantity of heat it can transport per unit change in temperature:
   i.e. its mass flow $\times$ specific heat capacity.

$$C = \frac{(\dot{m}C_P)_{\text{min}}}{(\dot{m}C_P)_{\text{max}}} = \frac{C_{\text{min}}}{C_{\text{max}}}$$

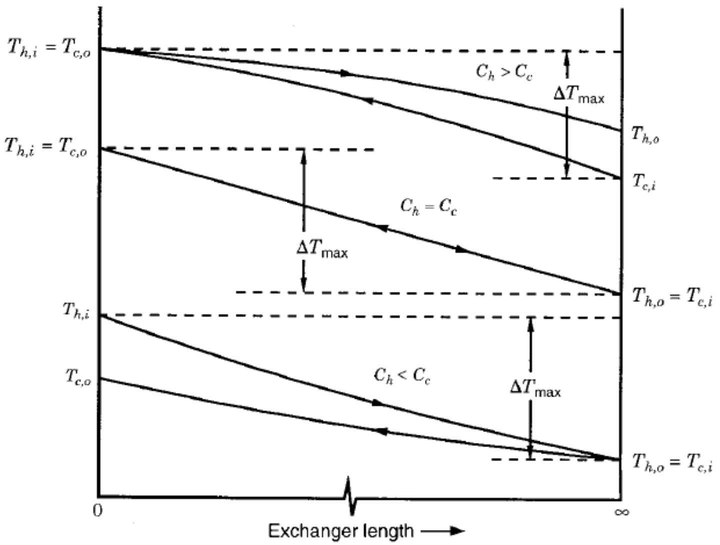
2. Thermal Effectiveness (ε):

$$\varepsilon = \frac{\text{actual heat transfer rate}}{\text{theoretical maximum heat transfer rate}} = \frac{Q}{Q_{\text{max}}} = \frac{Q}{C_{\text{min}}(T_{hi} - T_{ci})}$$

   The maximum theoretical heat transfer rate occurs in counterflow with infinite heat transfer surface area. It cannot occur in parallel flow because the exit temperature must be between the two inlet temperatures.

Effectiveness - NTU Method (contd..)

Maximum Possible Heat Transfer



Effectiveness - NTU Method (contd..)

The maximum theoretical heat transfer is given by:

$$Q_{\max} = (\dot{m}C_P)_{\min}(T_{hi} - T_{ci}) = C_{\min}(T_{hi} - T_{ci})$$

The actual heat transfer rate is given by:

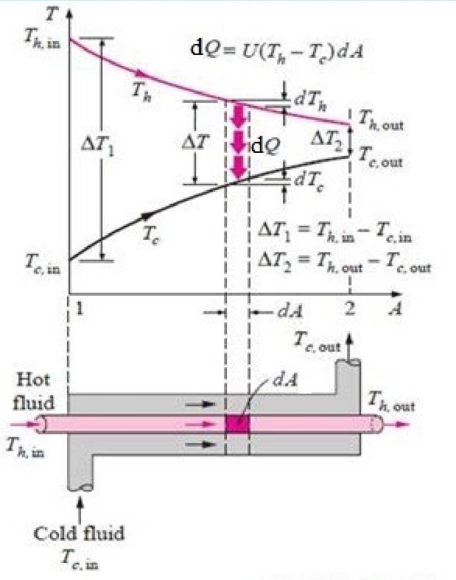
$$\begin{aligned} Q &= (\dot{m}C_P)_h(T_{hi} - T_{ho}) = C_h(T_{hi} - T_{ho}) \\ &= (\dot{m}C_P)_c(T_{co} - T_{ci}) = C_c(T_{co} - T_{ci}) \end{aligned}$$

Effectiveness - NTU Method (contd..)

3. Number of Transfer Units (NTU):

$$\text{NTU} = \frac{UA}{(mC_P)_{\text{min}}} = \frac{UA}{C_{\text{min}}}$$

Effectiveness - NTU Method for Cocurrent Exchanger



Effectiveness - NTU Method for Cocurrent Exchanger (contd..)

$$Q = UA\Delta T_m \tag{1}$$

For an elemental area dA,

$$dQ = U(\Delta T)(dA) \tag{2}$$

where $\Delta T = T_h - T_c$
From heat capacity relations, for the cold and hot fluids, we have

$$dQ = \dot{m}_c C_{P,c}\, dT_c = C_c\, dT_c \tag{3a}$$

$$dQ = -\dot{m}_h C_{P,h}\, dT_h = -\mathrm{C}_h\, dT_h \tag{3b}$$

where $C_c = \dot{m}_c C_{P,c}$, and $C_h = \dot{m}_h C_{P,h}$

Effectiveness - NTU Method for Cocurrent Exchanger (contd..)

$$\Delta T = T_h - T_c$$
$$d(\Delta T) = dT_h - dT_c$$

Substituting for dT_h and dT_c from Eqn.(3), we get

$$d(\Delta T) = -\frac{dQ}{C_h} - \frac{dQ}{C_c} = -dQ\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

Substituting for dQ from Eqn.(2), we get

$$d(\Delta T) = -U(\Delta T)(dA)\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

Rearranging,

$$\frac{d(\Delta T)}{\Delta T} = -U(dA)\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

Effectiveness - NTU Method for Cocurrent Exchanger (contd..)

For constant U,

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\int_0^A dA$$

$$\ln\frac{\Delta T_2}{\Delta T_1} = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \quad (4)$$

$$= -UAB$$

In terms of cold and hot fluid temperatures, we have,

$$\ln\left(\frac{T_{ho} - T_{co}}{T_{hi} - T_{ci}}\right) = -UAB$$

$$\frac{T_{ho} - T_{co}}{T_{hi} - T_{ci}} = e^{-UAB} \quad (5)$$

Effectiveness - NTU Method for Cocurrent Exchanger (contd..)

From energy balance between the cold and hot fluid, we get

$$C_h(T_{hi} - T_{ho}) = C_c(T_{co} - T_{ci})$$
$$T_{ho} = T_{hi} - \frac{C_c}{C_h}(T_{co} - T_{ci}) \tag{6}$$

Using Eqn.(6) in Eqn.(5), we get

$$\frac{T_{hi} - \frac{C_c}{C_h}(T_{co} - T_{ci}) - T_{co}}{T_{hi} - T_{ci}} = e^{-BAU}$$
$$\frac{(T_{hi} - T_{ci}) - \frac{C_c}{C_h}(T_{co} - T_{ci}) - T_{co} + T_{ci}}{T_{hi} - T_{ci}} = e^{-BAU}$$

Effectiveness - NTU Method for Cocurrent Exchanger (contd..)

$$\frac{(T_{hi} - T_{ci}) - \frac{C_c}{C_h}(T_{co} - T_{ci}) - (T_{co} - T_{ci})}{T_{hi} - T_{ci}} = e^{-BAU}$$

$$1 - \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}}\left(1 + \frac{C_c}{C_h}\right) = e^{-BAU}$$

$$1 - e^{-BAU} = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}}\left(1 + \frac{C_c}{C_h}\right) \quad (7)$$

From the definition of ε, we have

$$\varepsilon = \frac{Q}{C_{min}(T_{hi} - T_{ci})} \implies T_{hi} - T_{ci} = \frac{Q}{\varepsilon C_{min}} \quad (8)$$

For the cold fluid,

$$Q = C_c(T_{co} - T_{ci}) \implies T_{co} - T_{ci} = \frac{Q}{C_c} \quad (9)$$

Effectiveness - NTU Method for Cocurrent Exchanger (contd..)

Using Eqns.(8) and (9) in Eqn.(7), we get

$$1 - e^{-BAU} = \varepsilon \frac{C_{\min}}{C_c}\left(1 + \frac{C_c}{C_h}\right)$$

Rearranging, we get

$$\varepsilon = \frac{1 - e^{-BAU}}{\dfrac{C_{\min}}{C_c} + \dfrac{C_{\min}}{C_h}} \tag{10}$$

From Eqn.(4) we have,

$$B = \frac{1}{C_h} + \frac{1}{C_c}$$

And, from the definition of NTU $= N$ we have

$$N = \frac{AU}{C_{\min}}$$

Using these in Eqn.(10), we get

Effectiveness - NTU Method for Cocurrent Exchanger (contd..)

$$\varepsilon = \frac{1 - \exp\left[-\left(\frac{1}{C_h} + \frac{1}{C_c}\right) NC_{\text{min}}\right]}{\frac{C_{\text{min}}}{C_c} + \frac{C_{\text{min}}}{C_h}}$$

$$= \frac{1 - \exp\left[-N\left(\frac{C_{\text{min}}}{C_h} + \frac{C_{\text{min}}}{C_c}\right)\right]}{\frac{C_{\text{min}}}{C_c} + \frac{C_{\text{min}}}{C_h}}$$

From the definition of C. we have

$$C = \frac{C_{\text{min}}}{C_{\text{max}}}$$

Let $C_h = C_{\text{min}}$. Then,

$$\varepsilon = \frac{1 - \exp[(-N(1 + C)]}{1 + C}$$

Effectiveness - NTU Method for Cocurrent Exchanger (contd..)

If $C_c = C_{\min}$. Then also, we get

$$\varepsilon = \frac{1 - \exp[(-N(1 + C)]}{1 + C}$$

Hence, for the cocurrent heat exchanger, the relation between effectiveness (ε) and NTU (N) is given by

$$\boxed{\varepsilon = \frac{1 - \exp[(-N(1 + C)]}{1 + C}}$$

Effectiveness - NTU Method for Countercurrent Exchanger

Similar derivation can be done for countercurrent exchanger. However, the algebraic reduction is more involved. The final relation is given as

$$\boxed{\varepsilon = \frac{1 - \exp[-N(1-C)]}{1 - C\exp[-N(1-C)]}}$$

Solved Problem

Example 1: Effectiveness NTU Method

Water ($C_P = 4180$ J/kg.°C) is to be heated by solar-heated hot air ($C_P = 1010$ J/kg.°C) in a double-pipe counter flow heat exchanger. Air enters the heat exchanger at 90°C at a rate of 0.3 kg/s while water enters at 22°C at a rate of 0.1 kg/s. The overall heat transfer coefficient based on the inner side of the tube is given to be 80 W/m^2.°C. The length of the tube is 12 m and the inner diameter of the tube is 1.2 cm. Determine the outlet temperature of the water and the air. (AU-May-2017)

Solved Problems (contd..)

Solution:

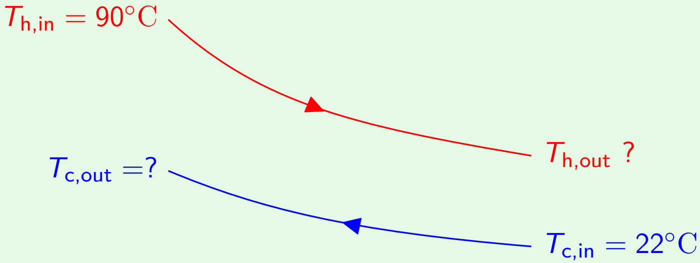
Data:

Cold fluid: water, $C_{P\text{c}} = 4180$ J/kg.$^\circ$C, $\dot{m}_\text{c} = 0.1$ kg/s

Hot fluid: air, $C_{P\text{h}} = 1010$ J/kg.$^\circ$C, $\dot{m}_\text{h} = 0.3$ kg/s

$A = \pi DL = 3.142 \times 0.012 \times 12 = 0.4524$ m^2

$U = 80$ W/m^2.$^\circ$C



Solved Problems (contd..)

Rate of heat transfer is given by

For the cold fluid (water):

$$Q = \dot{m}_\mathrm{c} C_{P\mathrm{c}}(T_\mathrm{c,out} - T_\mathrm{c,in}) = 0.1 \times 4180 \times (T_\mathrm{c,out} - 22) \tag{1}$$

For the hot fluid (air):

$$Q = \dot{m}_\mathrm{h} C_{P\mathrm{h}}(T_\mathrm{h,in} - T_\mathrm{h,out}) = 0.3 \times 1010 \times (90 - T_\mathrm{h,out}) \tag{2}$$

For the heat exchanger:

$$Q = UA\Delta T_\mathrm{lm} = 80 \times 0.4524 \times \left[\frac{(90 - T_\mathrm{c,out}) - (T_\mathrm{h,out} - 22)}{\ln\left(\frac{90 - T_\mathrm{c,out}}{T_\mathrm{h,out} - 22}\right)}\right] \tag{3}$$

Solved Problems (contd..)

The above 3 equations contain the unknowns Q, $T_{\mathrm{h,out}}$ and $T_{\mathrm{c,out}}$. Solving them involves a trial and error calculation.

The steps are:

1. Assume values for $T_{\mathrm{h,out}}$ or $T_{\mathrm{c,out}}$. Obtain the other ($T_{\mathrm{c,out}}$ or $T_{\mathrm{h,out}}$) from heat balance using Eqns.(1) or (2).
2. Obtain ΔT_{lm}, and calculate Q from Eqn.(3).
3. Using the value of Q, calculate $T_{\mathrm{c,out}}$ and $T_{\mathrm{h,out}}$ from Eqns.(1) and (2).
4. Compare the outlet temperatures determined in the step-3, with the values assumed in step-1.
5. If the calculated values of T are different from the assumed values, then repeat the calculations, until a specified convergence is achieved.

Clearly, such a computation is very tedious. The calculation may be simplified by using ε-NTU method.

Solved Problems (contd..)

Calculation steps as per ε-NTU method:

Step-1: (Calculation of C and N)

$$C_{\text{air}} = \dot{m}_{\text{h}} C_{P\text{h}} = 0.3 \times 1010 = 303 \text{ W}/^{\circ}\text{C} = C_{\text{min}}$$

$$C_{\text{water}} = \dot{m}_{\text{c}} C_{P\text{c}} = 0.1 \times 4180 = 418 \text{ W}/^{\circ}\text{C} = C_{\text{max}}$$

$$C = \frac{C_{\text{min}}}{C_{\text{max}}} = \frac{303}{418} = 0.725$$

and,

$$N = \text{NTU} = \frac{UA}{C_{\text{min}}} = \frac{80 \times 0.4524}{303} = 0.12$$

Solved Problems (contd..)

Step-2: (Calculation of ε)
For the countercurrent heat exchanger,

$$\varepsilon = \frac{1 - \exp\left[-N(1-C)\right]}{1 - C\exp\left[-N(1-C)\right]}$$
$$= \frac{1 - \exp[-0.12 \times (1 - 0.725)]}{1 - 0.725 \times \exp[-0.12 \times (1 - 0.725)]} = 0.109$$

Step-3: (Calculation of Q)

$$Q = \varepsilon C_{\text{min}}(T_{\text{h,in}} - T_{\text{c,in}})$$
$$= 0.109 \times 0.303 \times (90 - 22) = 2246 \text{ W}$$

Solved Problems (contd..)

Step-4: (Calculation of outlet temperatures)
From Eqns.(1) and (2), we get

$$T_{\mathrm{c,out}} = T_{\mathrm{c,in}} + \frac{Q}{\dot{m}_{\mathrm{c}} C_{P\mathrm{c}}} = 22 + \frac{2246}{0.1 \times 4180} = 27.4^{\circ}\mathrm{C}$$

$$T_{\mathrm{h,out}} = T_{\mathrm{h,in}} - \frac{Q}{\dot{m}_{\mathrm{h}} C_{P\mathrm{h}}} = 90 - \frac{2246}{0.3 \times 1010} = 82.6^{\circ}\mathrm{C}$$

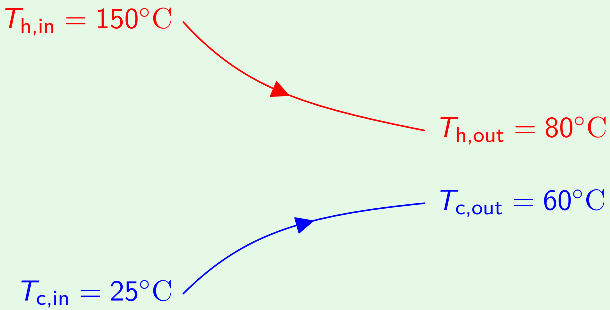
Solved Problem

Example 2: The Lowest Possible Temperature in a Parallel Flow Exchanger

The engine oil at $150°\mathrm{C}$ is cooled to $80°\mathrm{C}$ in a parallel flow heat exchanger by water entering at $25°\mathrm{C}$ and leaving at $60°\mathrm{C}$. Estimate the exchanger effectiveness and the number of transfer units. If the fluid flow rates and inlet conditions remain unchanged, work out the lowest temperature to which the oil may be cooled by increasing the length of the exchanger. (AU-Nov-2016)

Solved Problems (contd..)

Solution:



Solved Problems (contd..)

Effectiveness (ε):

$$\varepsilon = \frac{Q}{Q_{\text{max}}} = \frac{Q}{C_{\text{min}}(T_{\text{h,in}} - T_{\text{c,in}})} = \frac{Q}{C_{\text{min}}(150 - 25)} \quad (1)$$

To find the value of ε from the above equation, we need to find the fluid which is having C_{min}, and use the expression of $Q = C_{\text{min}}\Delta T_{\text{max}}$ From energy balance between the fluids,

$$Q = C_{\text{min}}\Delta T_{\text{max}} = C_{\text{max}}\Delta T_{\text{min}} \quad (2)$$

From the data given

$$\Delta T_{\text{oil}} = 150 - 80 = 70^{\circ}\text{C}$$
$$\Delta T_{\text{water}} = 60 - 25 = 35^{\circ}\text{C}$$

ΔT is higher for oil. Therefore, $C_{\text{min}} = C_{\text{oil}}$, and $C_{\text{max}} = C_{\text{water}}$

Solved Problems (contd..)

Hence, Eqn.(2) becomes,

$$Q = C_{\text{min}}(150 - 80) = C_{\text{max}}(60 - 25) \tag{3}$$

Using Eqn.(3), in Eqn.(1), we get

$$\varepsilon = \frac{C_{\text{min}}(150 - 80)}{C_{\text{min}}(150 - 25)} = 0.56$$

Heat capacity ratio (C):

$$C = \frac{C_{\text{min}}}{C_{\text{max}}} \tag{4}$$

From Eqn.(3), we have

$$\frac{C_{\text{min}}}{C_{\text{max}}} = \frac{60 - 25}{150 - 80} = 0.5 = \frac{C_{\text{oil}}}{C_{\text{water}}} \tag{5}$$

Solved Problems (contd..)

Number of Transfer Units (N):

$$N = \frac{UA}{C_{\min}}$$

From the definition of effectiveness, we have

$$\varepsilon = \frac{Q}{C_{\min}(T_{\mathrm{h,in}} - T_{\mathrm{c,in}})} = \frac{UA\,\Delta T_{\mathrm{lm}}}{C_{\min}(T_{\mathrm{h,in}} - T_{\mathrm{c,in}})} = N\frac{\Delta T_{\mathrm{lm}}}{(T_{\mathrm{h,in}} - T_{\mathrm{c,in}})}$$

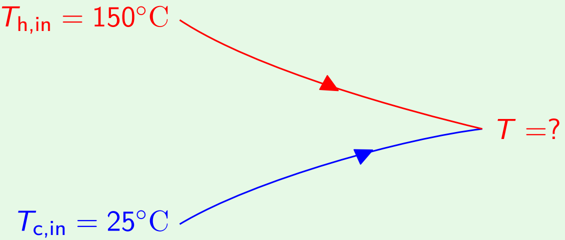
For the given parallel flow exchanger,

$$\Delta T_{\mathrm{lm}} = \frac{(150-25)-(80-60)}{\ln[(150-25)/(80-60)]} = 57.3^{\circ}\mathrm{C}$$

Hence,

$$N = \varepsilon\frac{T_{\mathrm{h,in}} - T_{\mathrm{c,in}}}{\Delta T_{\mathrm{lm}}} = 0.56 \times \frac{150-25}{57.3} = 1.22$$

Solved Problems (contd..)



From energy balance,

$$C_{\mathrm{oil}}(150 - T) = C_{\mathrm{water}}(T - 25) \implies \frac{150 - T}{T - 25} = \frac{C_{water}}{C_{oil}}$$

Using Eqn.(5) in above, we get

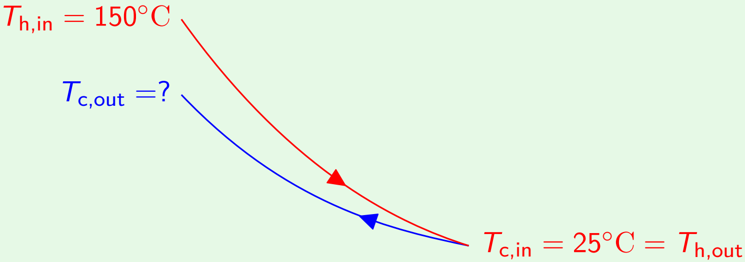
$$\frac{150 - T}{T - 25} = \frac{1}{0.5}$$

Solved Problems (contd..)

i.e.,

$$0.5 \times (150 - T) = T - 25 \qquad \Longrightarrow \qquad T = 66.7^\circ\mathrm{C}$$

Extension of the problem:
With infinite heat transfer area, if the arrangement were countercurrent, then oil (the fluid with C_{min}) will exit at the temperature of $25^\circ\mathrm{C}$.



Solved Problems (contd..)

The exit temperature of water ($T_{c,out}$) is obtained as below:

$$C_{water}(T_{c,out} - 25) = C_{oil}(150 - 25) \qquad \frac{T_{c,out} - 25}{150 - 25} = \frac{C_{oil}}{C_{water}}$$

From the previous calculations, we have

$$\frac{C_{oil}}{C_{water}} = 0.5$$

hence,

$$\frac{T_{c,out} - 25}{150 - 25} = 0.5 \qquad \Longrightarrow \qquad T_{c,out} = 87.5$$

Questions for Practice

1. Hot water at 2.5 kg/s and 100°C enters a concentric tube heat exchanger having a total area of 23 m^2. Cold water at 20°Centers at 5 kg/s and the overall heat transfer coefficient is 1000 W/m^2.K. Find the heat transfer rate and the outlet temperature of hot and cold fluids. (AU-May-2017)
   Assume:
   (i) Counter flow in heat exchanger.
   (ii) Hot and cold water C_P is 4200 J/kg.K.
2. A heat exchanger has a mean overall heat transfer coefficient of 420 W/m^2.K based on the side whose surface area is 100 m^2. Find the outelet temperatures of hot and cold fluids for both counter and parallel flow operations from the data given below: (AU-Nov-2007)

	Hot fluid	Cold fluid
Inlet temperature (°C)	700	100
Mass flow rate (kg/min)	1000	1200
Heat capacity (kJ/kg.K)	3.6	4.2