Heat Transfer

Heat Exchangers - LMTD Method

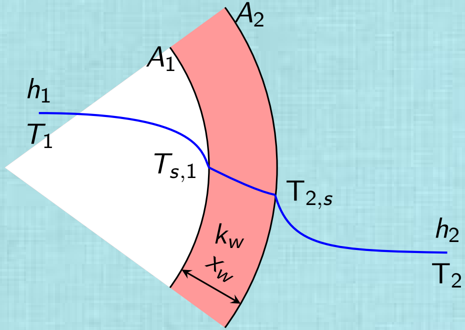
Dr. M. Subramanian

Department of Chemical Engineering
SSN College of Engineering

September 25, 2019

Overall Heat Transfer Coefficient (U)

$$Q = \frac{\Delta T}{R} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T$$



Overall Heat Transfer Coefficient (contd..)

$$T_1 - T_{1,s} = \frac{Q}{h_1 A_1} \qquad T_{1,s} - T_{2,s} = \frac{x_w Q}{k_w A_m} \qquad T_{2,s} - T_2 = \frac{Q}{h_2 A_2}$$

Adding the numerator and denominator separately, we get

$$T_1 - T_2 = \Delta T = Q\left[\frac{1}{h_1 A_1} + \frac{x_w}{k_w A_m} + \frac{1}{h_2 A_2}\right]$$

From the relation $Q = U_1 A_1 \Delta T$, we get

$$\frac{Q}{U_1 A_1} = Q\left[\frac{1}{h_1 A_1} + \frac{x_w}{k_w A_m} + \frac{1}{h_2 A_2}\right]$$

i.e.,

$$\frac{1}{U_1} = \frac{1}{h_1} + \frac{x_w}{k_w}\frac{A_1}{A_m} + \frac{1}{h_2}\frac{A_1}{A_2}$$

Similarly, we can also write

$$\frac{1}{U_2} = \frac{1}{h_2} + \frac{x_w}{k_w}\frac{A_2}{A_m} + \frac{1}{h_1}\frac{A_2}{A_1}$$

Overall Heat Transfer Coefficient (contd..)

For thin-walled tubes, where $A_1 \approx A_2 \approx A_m$, we can write,

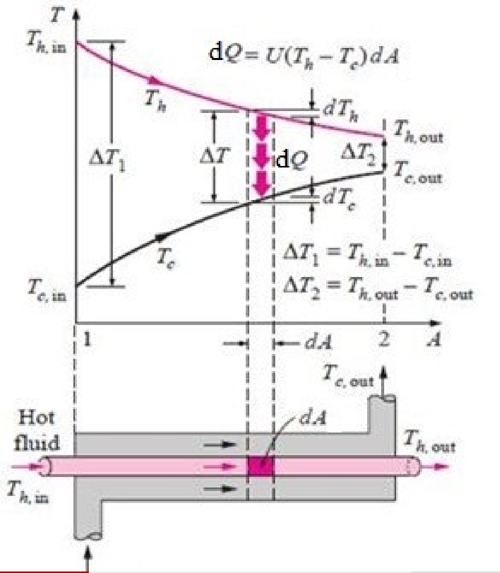
$$\frac{1}{U} = \frac{1}{h_1} + \frac{x_w}{k_w} + \frac{1}{h_2}$$

For the case of highly conducting wall, we can write the above as,

$$\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2}$$

LMTD for Heat Exchanger Analysis

Cocurrent Flow



LMTD for Heat Exchanger Analysis (contd..)

Cocurrent Flow

$$Q = UA\Delta T_m \tag{1}$$

For an elemental area dA,

$$dQ = U(\Delta T)(dA) \tag{2}$$

where $\Delta T = T_h - T_c$
From heat capacity relations, for the cold and hot fluids, we have

$$dQ = \dot{m}_c C_{P,c}\, dT_c = C_c\, dT_c \tag{3a}$$

$$dQ = -\dot{m}_h C_{P,h}\, dT_h = -\mathrm{C}_h\, dT_h \tag{3b}$$

where $C_c = \dot{m}_c C_{P,c}$, and $C_h = \dot{m}_h C_{P,h}$

LMTD for Heat Exchanger Analysis (contd..)

Cocurrent Flow

$$\Delta T = T_h - T_c$$
$$d(\Delta T) = dT_h - dT_c$$

Substituting for dT_h and dT_c from Eqn.(3), we get

$$d(\Delta T) = -\frac{dQ}{C_h} - \frac{dQ}{C_c} = -dQ\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

Substituting for dQ from Eqn.(2), we get

$$d(\Delta T) = -U(\Delta T)(dA)\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

Rearranging,

$$\frac{d(\Delta T)}{\Delta T} = -U(dA)\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

LMTD for Heat Exchanger Analysis (contd..)

Cocurrent Flow

For constant U,

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\int_0^A dA$$

$$\ln\frac{\Delta T_2}{\Delta T_1} = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \tag{4}$$

For the cold and hot fluids, we have

$$Q = C_c(T_{c,\mathrm{out}} - T_{c,\mathrm{in}}) = C_c(T_{c2} - T_{c1})$$

$$Q = C_h(T_{h,\mathrm{in}} - T_{h,\mathrm{out}}) = C_h(T_{h1} - T_{h2})$$

From these we get

$$\frac{1}{C_c} = \frac{T_{c2} - T_{c1}}{Q} \tag{5a}$$

$$\frac{1}{C_h} = \frac{T_{h1} - T_{h2}}{Q} \tag{5b}$$

LMTD for Heat Exchanger Analysis (contd..)

Cocurrent Flow

Using Eqn.(5) in Eqn.(4), we get

$$\ln\frac{\Delta T_2}{\Delta T_1} = -UA\left(\frac{T_{h1} - T_{h2}}{Q} + \frac{T_{c2} - T_{c1}}{Q}\right)$$

$$= -\frac{UA}{Q}\left[(T_{h1} - T_{c1}) - (T_{h2} - T_{c2})\right] = \frac{UA}{Q}(\Delta T_2 - \Delta T_1)$$

$$\implies \quad Q = UA\frac{\Delta T_2 - \Delta T_1}{\ln\dfrac{\Delta T_2}{\Delta T_1}} \tag{6}$$

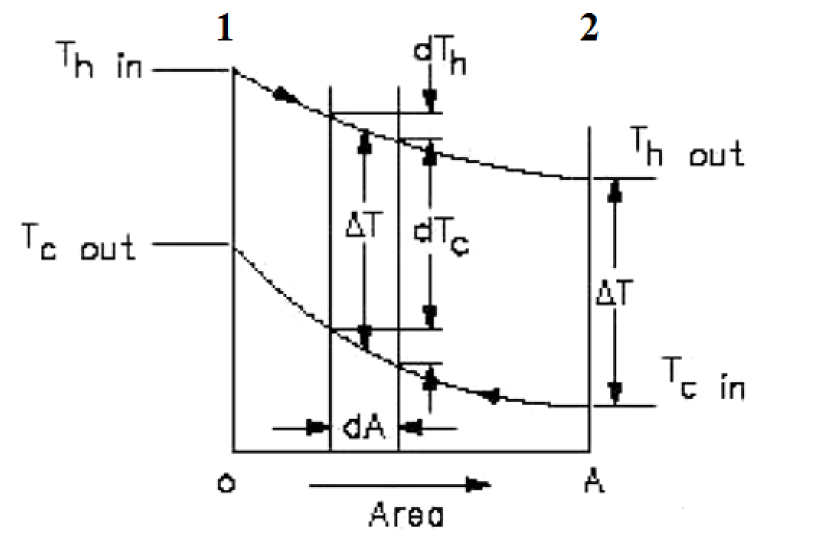
Comparing Eqns.(1) and (6), we get

$$\boxed{\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{\ln\dfrac{\Delta T_2}{\Delta T_1}} = \Delta T_{\mathrm{lm}} = \mathrm{LMTD}}$$

LMTD = Log Mean Temperature Difference

LMTD for Heat Exchanger Analysis (contd..)

Counter-current Flow



LMTD for Heat Exchanger Analysis (contd..)

Counter-current Flow

$$Q = UA\Delta T_m \tag{1}$$

For an elemental area dA,

$$dQ = U(\Delta T)(dA) \tag{2}$$

where $\Delta T = T_h - T_c$.
From heat capacity relations, for the cold and hot fluids, we have

$$dQ = -C_c \, dT_c \tag{3a}$$

$$dQ = -\mathrm{C}_h \, dT_h \tag{3b}$$

where $C_c = \dot{m}_c C_{P,c}$, and $C_h = \dot{m}_h C_{P,h}$

LMTD for Heat Exchanger Analysis (contd..)

Counter-current Flow

$$\Delta T = T_h - T_c$$
$$d(\Delta T) = dT_h - dT_c$$

Substituting for dT_h and dT_c from Eqn.(3), we get

$$d(\Delta T) = -\frac{dQ}{C_h} + \frac{dQ}{C_c} = dQ\left(\frac{1}{C_c} - \frac{1}{C_h}\right)$$

Substituting for dQ from Eqn.(2), we get

$$d(\Delta T) = U(\Delta T)(dA)\left(\frac{1}{C_c} - \frac{1}{C_h}\right)$$

Rearranging,

$$\frac{d(\Delta T)}{\Delta T} = U(dA)\left(\frac{1}{C_c} - \frac{1}{C_h}\right)$$

LMTD for Heat Exchanger Analysis (contd..)

Counter-current Flow

By continuing the derivation similar to co-current, we get the final relation as

$$Q = UA\Delta T_{\mathrm{lm}}$$

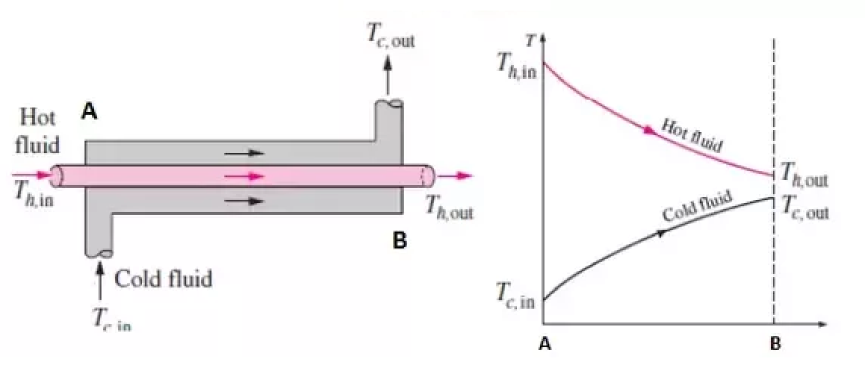
where

$$\boxed{\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{\ln \dfrac{\Delta T_2}{\Delta T_1}} = \Delta T_{\mathrm{lm}} = \mathrm{LMTD}}$$

LMTD = Log Mean Temperature Difference
This final relation is same as that cocurrent. But ΔT_1 and ΔT_2 are not the same.

Temperature Profiles

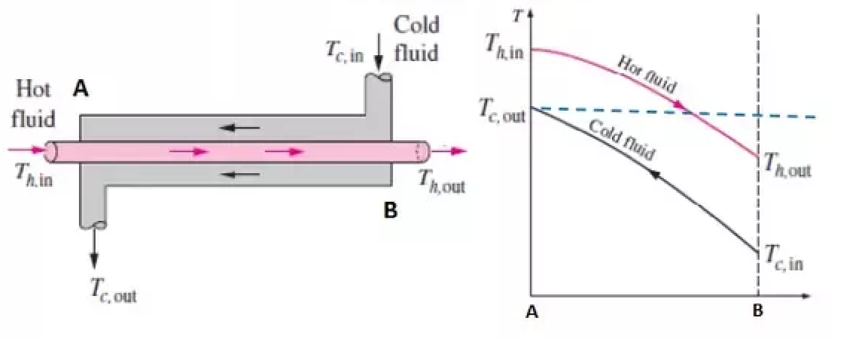
Co-Current Flow



$T_{c,out}$, out cannot exceed $T_{h,out}$.

Temperature Profiles

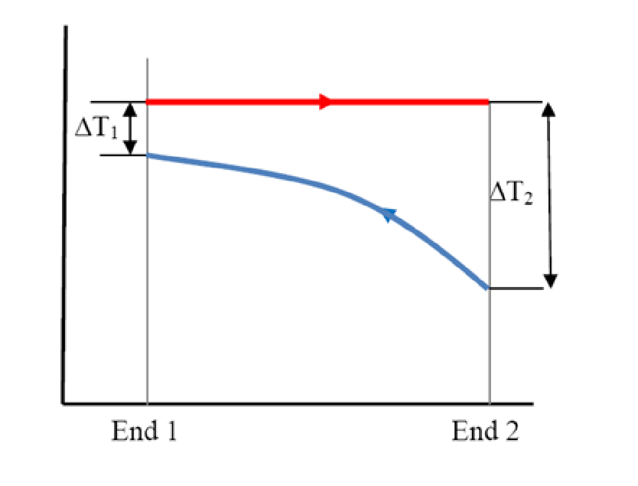
Counter Current Flow



$T_{\text{c,out}}$, out can exceed $T_{\text{h,out}}$.

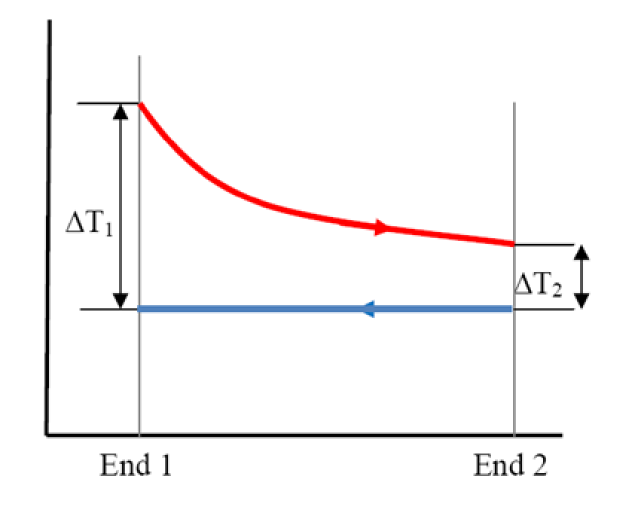
Temperature Profiles with Phase Change

Condenser



Temperature Profiles with Phase Change

Boiler



Comparison between Co-current and Counter-current

- To achieve greater heat recovery, a counter-current design is preferred to that of a co-current design.
- If we want to ensure that the temperature of the cold fluid never exceeds a particular temperature, then co-current exchanger designs are advantageous.
- Suppose one of the two interacting fluids is undergoing a phase change due to the heat transfer (e.g.: condensation of saturated steam), then both designs are identical.

Solved Problem

Example 1: Comparison of ΔT of Cocurrent and Countercurrent Flows

Hot water (0.01 m^3/min) enters the tube side of a cocurrent shell and tube heat exchanger at 80°C and leaves at 50°C. Cold oil (0.05 m^3/min) of density 800 kg/m^3 and specific heat of 2 kJ/(kg.K) enters at 20°C. The log mean temperature difference in $^\circ$C is approximately (G-2004-61)

(a) 32 (b) 37 (c) 45 (d) 50

What is the log mean temperature difference if the flow is countercurrent?

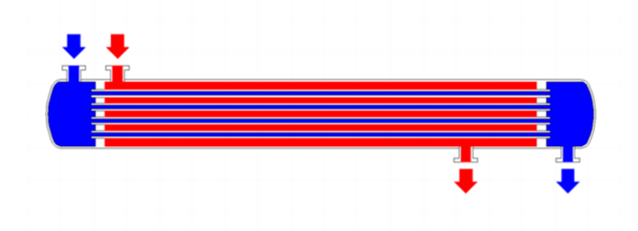
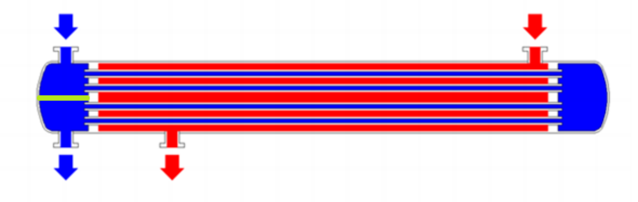


Figure : 1-1 Exchanger



Heat Exchangers

Flow Arrangement (contd..)

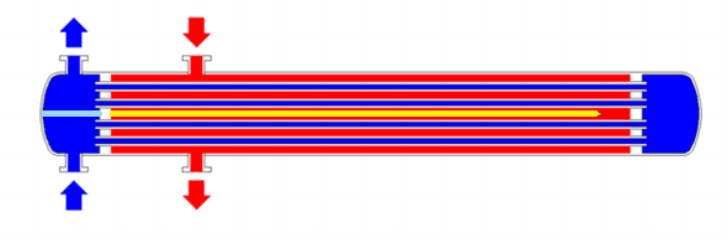
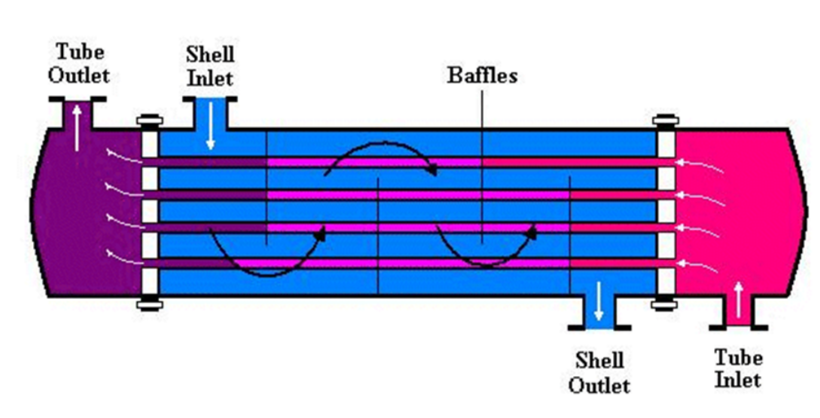


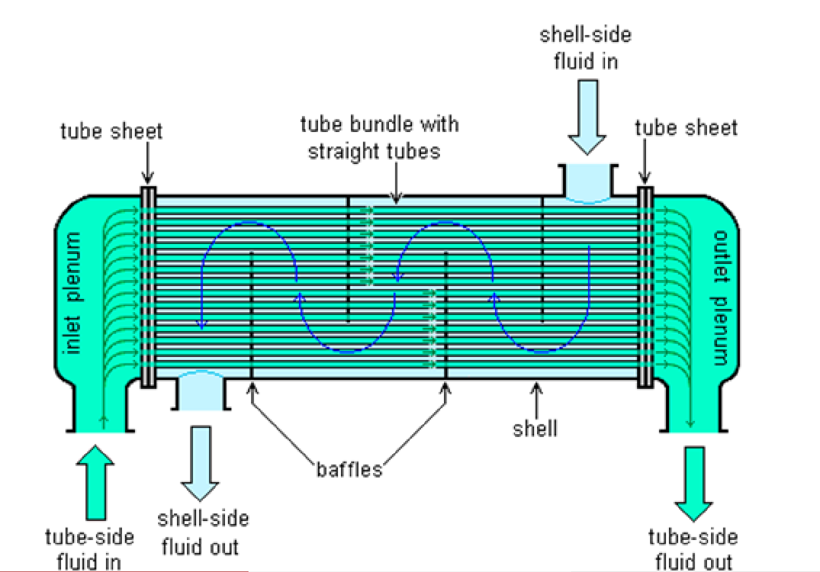
Figure : 2-2 Exchanger

Heat Exchangers

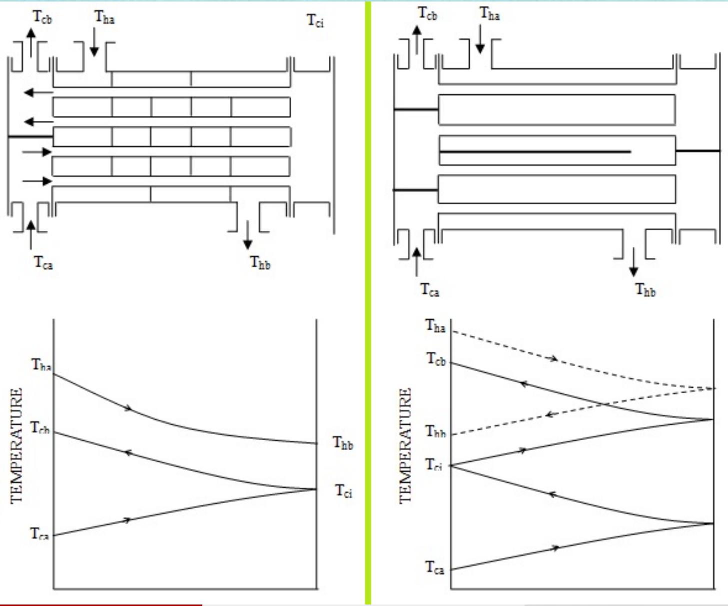


Heat Exchangers

Parts of Heat Exchanger



Temperature Profile of Multipass Exchangers



Correction Factor for LMTD

In most shell and tube exchangers the flow will be a mixture of co-current, counter-current and cross flow. The usual practice in the design of shell and tube exchangers is to estimate the "true temperature difference" from the logarithmic mean temperature by applying a correction factor (F; or F_t, F_T) to allow for the departure from true counter-current flow:

$$\Delta T_{\text{corr}} = F \, \Delta T_{\text{lm}}$$

F depends on the geometry of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams.

Correction Factor for LMTD (contd..)

Charts for correction factor (F) are available for commonly used heat exchanger configurations. In these figures, the abscissa is a dimensionless ratio P, defined as

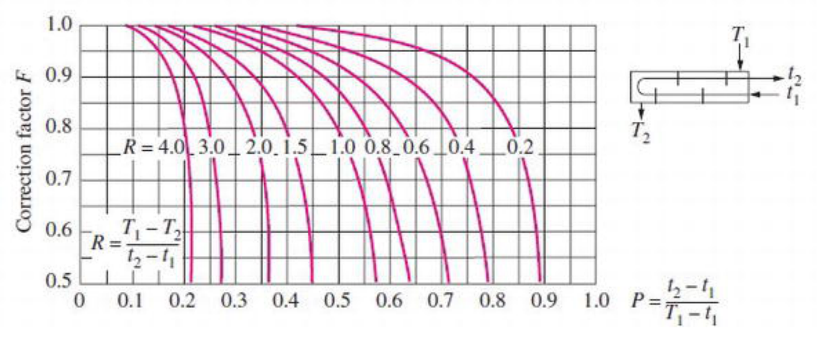
$$P = \frac{t_2 - t_1}{T_1 - t_1}$$

where T represents the shell-side temperature, t to the tube-side temperature, and subscripts 1 and 2, respectively to the inlet and outlet conditions. The parameter R appearing on the curves is defined as

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}C_P)_{\text{tube side}}}{(\dot{m}C_P)_{\text{shell side}}}$$

Correction Factor for LMTD (contd..)

F for 1-2 Exchanger



Correction Factor for LMTD (contd..)

- Generally F is less than unity for cross-flow and multipass arrangements; it is unity for true countercurrent flow heat exchanger.
- F represents the degree of departure of the true mean temperature difference from the LMTD for the counterflow.
- It should be noted that in case of condensation or evaporation the correction factor becomes unity ($F = 1$).
- While designing a heat exchanger, the rule of thumb is that the F should not be less than 0.8.

Fouling Factor

Over a time period of heat exchanger operation the surface of the heat exchanger may be coated by the various deposits present in the flow system. These deposits are known as scales. These scales provide another resistance and usually decrease the performance of the heat exchangers. The overall effect is usually represented by dirt factor or fouling factor, or fouling resistance, R_f which must be included for the calculation of overall heat transfer coefficient.

$$\boxed{R_f = \frac{1}{U_{\text{dirty}}} - \frac{1}{U_{\text{clean}}}}$$

Thus to determine the R_f, it is very important to know U_{clean} for the new heat exchanger. The U_{clean} data must be kept securely to obtain the R_f, at any time of the exchanger's life.

Fouling Factor

Representative fouling factors (thermal resistance due to fouling for a unit surface area)

(*Source:* Tubular Exchange Manufacturers Association.)

Fluid	R_f, $m^2 \cdot °C/W$
Distilled water, sea-water, river water, boiler feedwater:	
Below 50°C	0.0001
Above 50°C	0.0002
Fuel oil	0.0009
Steam (oil-free)	0.0001
Refrigerants (liquid)	0.0002
Refrigerants (vapor)	0.0004
Alcohol vapors	0.0001
Air	0.0004

Overall Heat Transfer Coefficient including Fouling Coefficients

$$\boxed{\frac{1}{U_1 A_1} = \frac{1}{h_1 A_1} + \frac{1}{h_{1,d} A_1} + \frac{x_w}{k_w A_m} + \frac{1}{h_{2,d} A_2} + \frac{1}{h_2 A_2}}$$

where $h_{1,d}$ and $h_{2,d}$ are fouling coefficients.

Questions for Practice

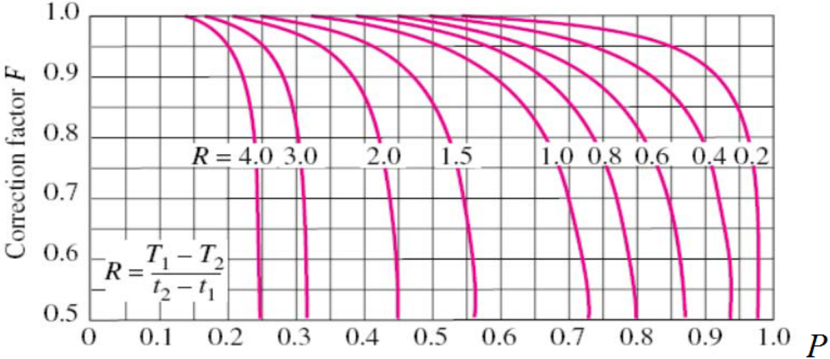
Ex-1

An organic vapor is condensing on the outer surface of a 5 m long copper ($k = 400$ W/m.K) tube having 25 mm OD and 20 mm ID. The heat transfer coefficient of the vapor is 1500 W/m^2.K. The coolant enters at 300 K and leaves at 325 K. The coolant side heat transfer coefficient is 2500 W/m^2.K and the fouling coefficients on the inner and outer surfaces are 2000 W/m^2.K and 4000 W/m^2.K respectively. The vapor condenses at 355 K. Determine the overall heat transfer coefficient and the heat transfer rate. (AU-Nov-2007)

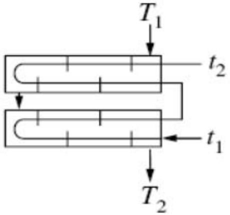
Questions for Practice

Ex-2

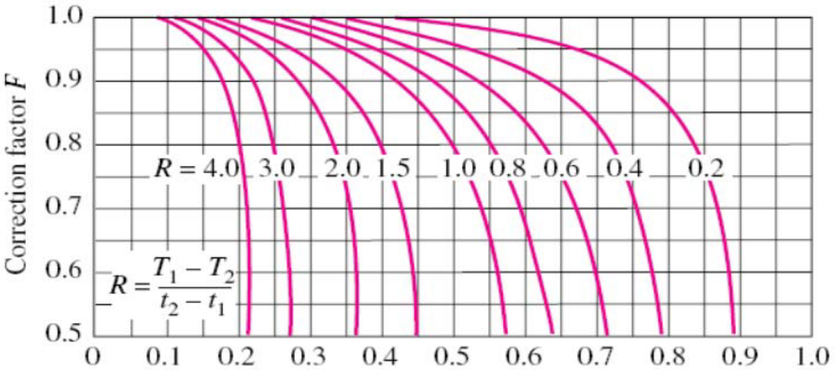
A shell and tube heat exchanger with 2-shell passes and 8-tube passes is used to heat ethyl alcohol ($C_P = 2670$ J/kg.$^\circ$C) in the tubes from 25°C to 70°C at a rate of 2.1 kg/s. The heating is to be done by water ($C_P = 4190$ J/kg.$^\circ$C) that enters the shell side at 95°C and leaves at 45°C. If the overall heat transfer coefficient is 950 W/m^2.$^\circ$C, determine the heat transfer and the surface area of the heat exchanger. (AU-Nov-2016)



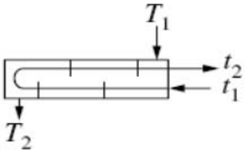
Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes



$$P = \frac{t_2 - t_1}{T_1 - t_1}$$



One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes



$$P = \frac{t_2 - t_1}{T_1 - t_1}$$

Questions for Practice

Ex-3

A chemical plant produces 300 metric tons of sulphuric acid per day. The acid is to be cooled from 333 K to 313 K by 500 metric tons of water per day, which has a initial temperature of 288 K. A counter flow cooler consisting of concentric pipes 12.5 mm thick is to be used. The inner pipe through which the acid flow is 75 mm bore and outer one 125 mm bore. The outside diameter of the inner pipe is 100 mm. The physical properties of the fluid the mean temperature are as follows:

	Acid	Water
Density, kg/m^3	1800	998.2
Heat capacity, kJ/kg.K	1.465	4.187
Thermal conductivity, W/m.K	0.302	0.669
Viscosity, kg/m.s	0.0112	0.0011

Thermal conductivity of the pipe material is 46.52 W/m.K. Use Dittus-Boelter equation for the calculation of h. Also calculate the length of pipe required. (AU-Nov-2014)