Heat Transfer

Convection - Analogies between Heat and Momentum Transfer

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September 25, 2019

Use for Analogies: It is comparatively easy to experimentally/theoretically evaluate the momentum transport under various conditions. However, the heat transport is not so easy to find out. Therefore, we will learn different analogies to find the heat transport relations.

Laminar Flow

Momentum transfer:

$$\frac{\tau}{\rho} = \nu \frac{dv}{dy}$$

Heat transfer:

$$\frac{q}{\rho C_P} = -\alpha \frac{dT}{dy}$$

where y is the distance measured from the tube wall. ν and α are molecular diffusivities (laminar).

Turbulent Flow

Momentum transfer:

$$\frac{\tau}{\rho} = (\nu + \varepsilon_m)\frac{dv}{dy}$$

Heat transfer:

$$\frac{q}{\rho C_P} = -(\alpha + \varepsilon_h)\frac{dT}{dy}$$

where y is the distance measured from the tube wall. ε is eddy diffusivity (turbulent).

Reynolds assumed that the entire flow field consisted of a single zone of highly turbulent region. That is, he neglected the presence of the viscous sublayer and the buffer layer. In such a turbulent core,

$$\nu \ll \varepsilon_m \qquad \text{and } \alpha \ll \varepsilon_h$$

In addition, he assumed that the turbulent diffusivities are equal.

$$\varepsilon_m = \varepsilon_m = \varepsilon$$

With the above assumptions,

$$\frac{\tau}{\rho} = \varepsilon \frac{dv}{dy}$$
$$\frac{q}{\rho C_P} = -\varepsilon \frac{dT}{dy}$$

Reynolds Analogy (contd..)

Combining the above two equations, we get

$$dT = -\frac{q}{\tau C_P} dv$$

Integration limits:

Wall conditions:	$T = T_w$	and	$v = 0$
Bulk stream conditions:	$T = T_m$	and	$v = v_m$

Assuming q/τ remains constant,

$$\int_{T_w}^{T_m} dT = -\frac{q}{\tau C_P} \int_0^{v_m} dv$$

$$T_w - T_m = \frac{q v_m}{\tau C_P} \qquad (1)$$

Reynolds Analogy (contd..)

By definition,

$$h = \frac{q}{T_w - T_m}$$

and

$$f = \frac{\tau}{\rho v_m^2/2}$$

Therefore, Eqn.(1) becomes,

$$\boxed{\frac{h}{\rho C_P v_m} = \text{St} = \frac{f}{2}}$$

This result is known as Reynolds analogy for momentum and heat transfer in fully developed turbulent flow in a pipe. It is valid for Pr ≈ 1, and negligible pressure gradient ($dp/dx \approx 0$).

Prandtl Analogy

Prandtl assumed that the flow field consisted of two layers, a viscous sublayer where the molecular diffusivities are dominant, that is,

$$\varepsilon_m \ll \nu \qquad \text{and} \qquad \varepsilon_h \ll \alpha$$

and a turbulent zone where the turbulent diffusivities are dominant, that is

$$\nu \ll \varepsilon_m \qquad \text{and} \qquad \alpha \ll \varepsilon_h \qquad \text{and} \qquad \varepsilon_m = \varepsilon_h = \varepsilon$$

Using the above in the equations for momentum and heat transfer in each layer, and using the definitions of h and f, we get

$$\boxed{\mathrm{St} = \frac{h}{\rho C_P v_m} = \frac{f}{2}\left(\frac{1}{1 + 5\sqrt{f/2}(\mathrm{Pr} - 1)}\right)}$$

This reduces to Reynolds analogy for $\mathrm{Pr} = 1$.

Von Karman Analogy

Von Karman extended Prandtl's analogy by separating the flow field into three distinct layers: a viscous sublayer, a buffer layer, and a turbulent core.
In the buffer layer, molecular and eddy diffusivities are assumed to be of the same order of magnitude.

$$\boxed{\mathrm{St} = \frac{h}{\rho C_P v_m} = \frac{f}{2}\left(\frac{1}{1 + 5\sqrt{f/2}\,\{(\mathrm{Pr}-1) + \ln\left[(5\mathrm{Pr}+1)/6\right]\}}\right)}$$

The Reynolds analogy does not always give satisfactory results. Thus, Chilton and Colburn experimentally modified the Reynolds' analogy. The empirically modified Reynolds' analogy is known as Chilton-Colburn analogy.

$$\text{St}\ \text{Pr}^{2/3} = \text{ Colburn } j\text{-factor } = j_H = \frac{f}{2}$$

Valid for: $0.6 < \text{Pr} < 60$.
Laminar flows: valid for $dP/dx \approx 0$
Turbulent flows: generally valid without restriction on dP/dx.

Advantages of Analogies

The advantage of the analogy lies in that the h may not be available for certain geometries/situations however, for which f value may be available as it is easier to perform momentum transport experiments and then to calculate the f. Thus by using the analogies the h may be found out without involving into the exhaustive and difficult heat transfer experiments.