UCH1603 Process Dynamics and Control

Bode Stability Criteria

Dr. M. Subramanian

Department of Chemical Engineering
SSN College of Engineering
subramanianm@ssn.edu.in

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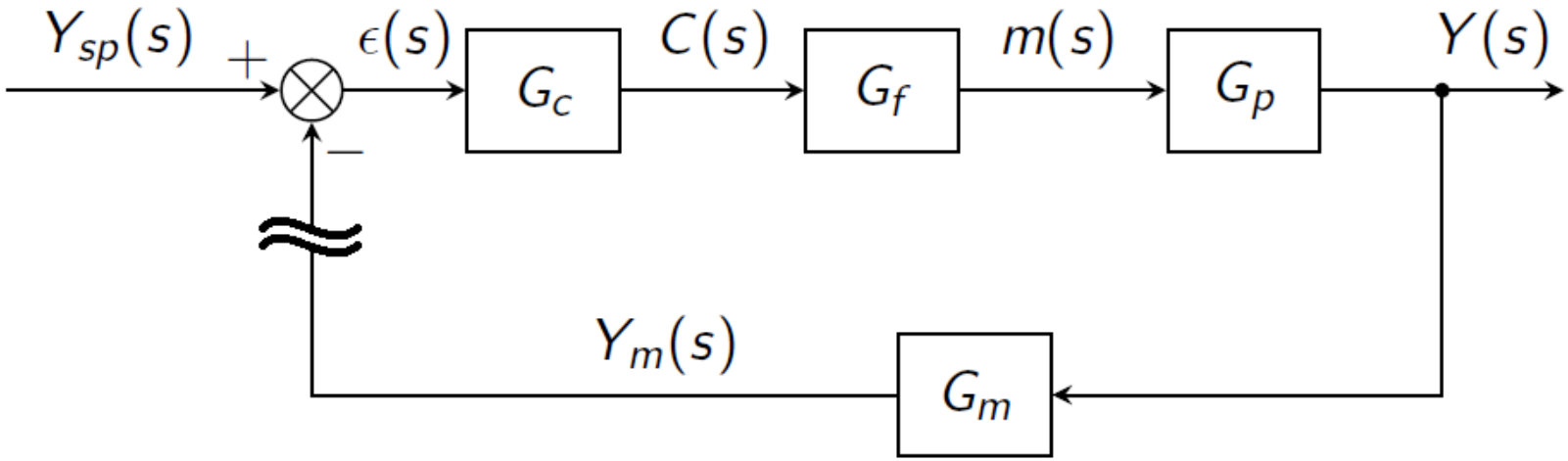


m - manipulated variable

Introduction

Starting from initial steady state, let us consider that the set point is varied sinusoidally with, $Y_{sp}(t) = \sin(\omega t)$, for a long period of time. Assume that during this period the measured output, Y_m, is disconnected so that the feedback loop is broken before the comparator.

After the initial transient dies out, Y_m will oscillate at the excitation frequency ω because the response of a linear system to a sinusoidal input is a sinusoidal output at the same frequency.

If the input frequency is corresponding to a phase shift of -180° for the open-loop, then for a setpoint of $Y_{sp}(t) = \sin(\omega t)$, we get $Y_m(t) = \sin(\omega t - 180^\circ) = -\sin(\omega t)$.

The frequency where the phase lag is equal to 180° is called the crossover frequency and is denoted by ω_{co}.

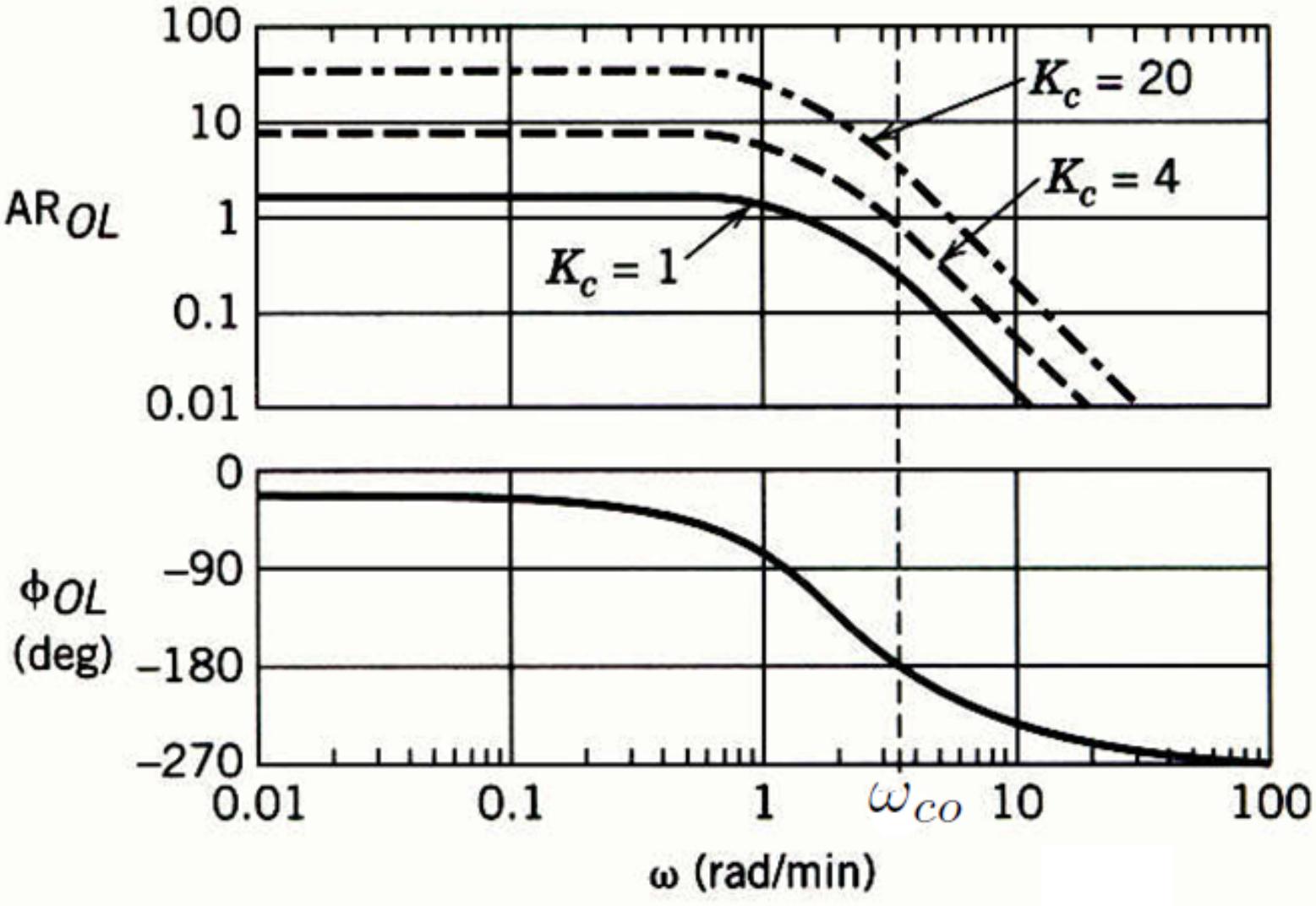
Introduction (contd..)

Suppose that two events occur simultaneously: (i) the set point is set to zero and, (ii) Y_m is reconnected. Under these conditions, the comparator inverts the sign of the Y_m, which now plays the same role as that played by the setpoint in the 'open-loop'. Now the error (ϵ) remains the same.

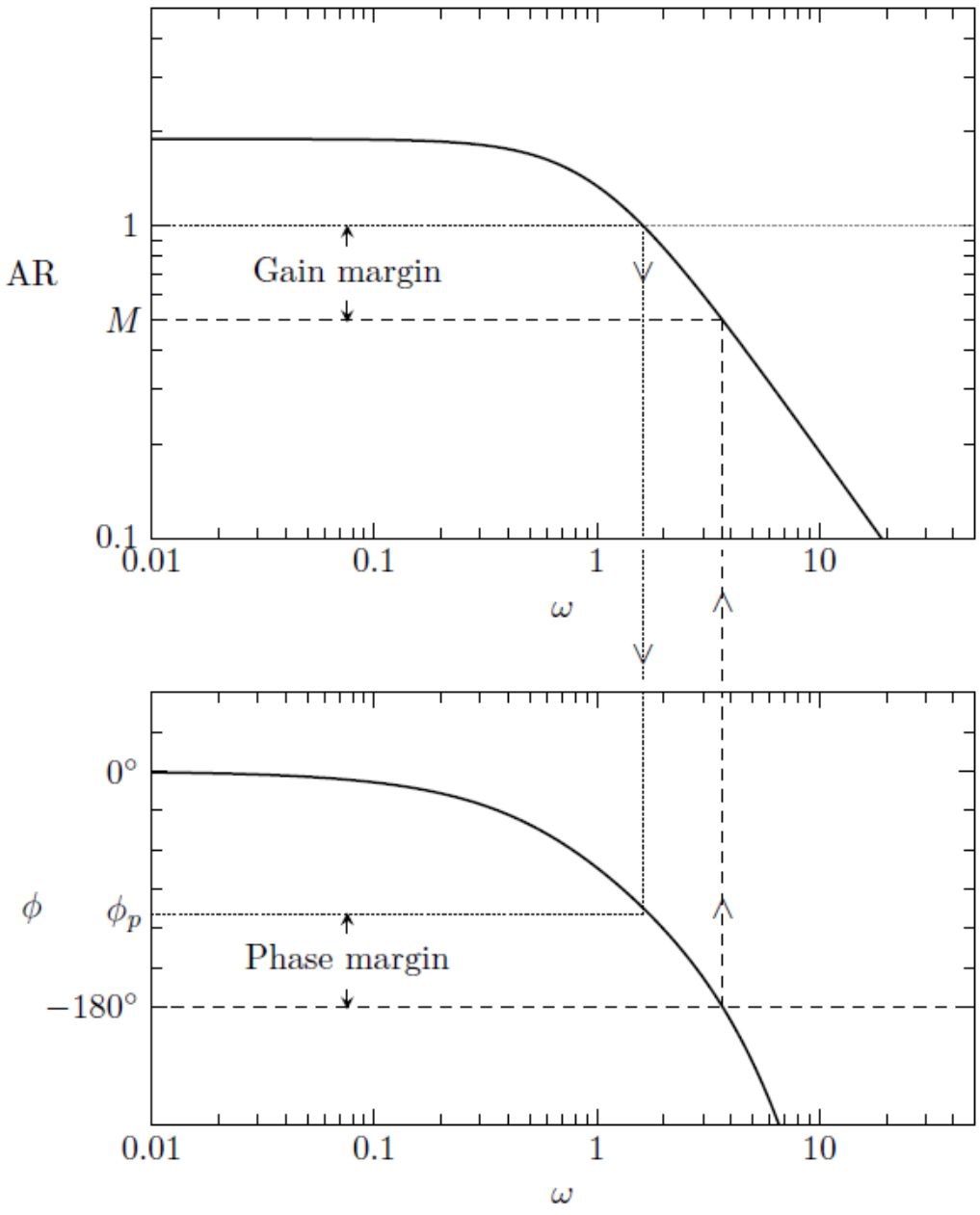
- Theoretically, the response of the system will continue to oscillate with constant amplitude, for AR $= 1$, despite the fact that both the load and setpoint do not change.
- If AR > 1 when $\phi = -180^\circ$, the closed-loop system will exhibit oscillations with ever-increasing amplitude leading to an unstable system.
- On the contrary, if AR < 1, when $\phi = -180^\circ$, the oscillating response of the closed-loop will exhibit a continuously decreasing amplitude, leading to eventual dying out of the oscillation.

Bode Stability Criterion

A feedback control system is unstable if the ‘Amplitude Ratio’ of the corresponding open-loop transfer function is larger than 1 at the crossover frequency.


100
10
1
0.1
0.01
AR$_{OL}$
K_c = 20
K_c = 4
K_c = 1
0
−90
−180
−270
Φ_{OL}
(deg)
0.01
0.1
1
ω_{co}
10
100
ω (rad/min)

Gain Margin and Phase Margin



$$\text{Gain margin} = \frac{1}{M}$$

$$> 1 \implies \text{Stable system}$$

$$\text{Phase margin} = 180 - \phi_p$$

Gain Margin and Phase Margin (contd..)

- Gain margin larger than 1 is a safety factor for stable operation. Typical gain margins are in the range 1.4 to 3.
- Typical phase margins used by the designers are larger than 30°.

Inapplicability of Bode Criteria