# UCH1603 Process Dynamics and Control

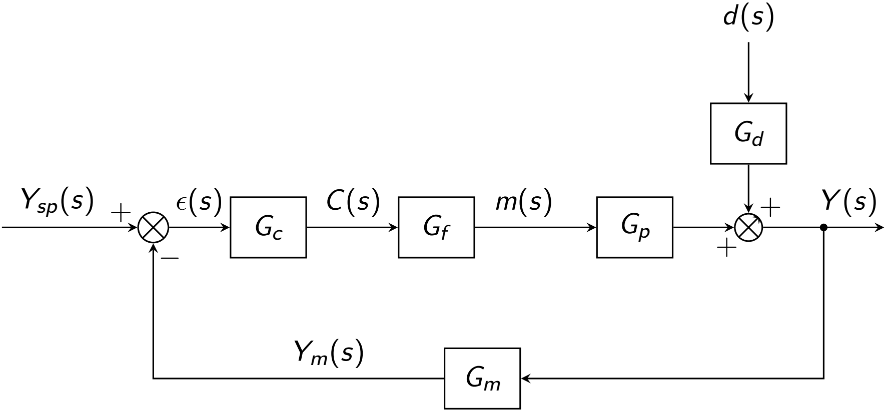
## Closed Loop Response

Dr. M. Subramanian

Department of Chemical Engineering
SSN College of Engineering
subramanianm@ssn.edu.in

March 5, 2021

# Closed Loop System



# Open Loop Response

Let us consider a first order system, for which the uncontrolled response is given by

$$\tau_p \frac{dY}{dt} + Y = K_p m + K_d d$$

Taking Laplace transforms,

$$\boxed{Y(s) = \frac{K_p}{\tau_p s + 1} m(s) + \frac{K_d}{\tau_p s + 1} d(s)}$$

i.e.,

$$Y(s) = G_p m(s) + G_d d(s)$$

where the transfer functions $G_p$ and $G_d$ are:

$$G_p = \frac{K_p}{\tau_p s + 1} \qquad \text{and} \qquad G_d = \frac{K_d}{\tau_p s + 1}$$

# Effect of Proportional Control

For the typical closed loop system, the response is given by

$$Y(s) = \frac{G_p G_f G_c}{1 + G_p G_f G_c G_m} Y_{sp}(s) + \frac{G_d}{1 + G_p G_f G_c G_m} d(s)$$

For proportional controller,

$$G_c = K_c$$

For simplicity, let us consider the case with $G_f = G_m = 1$. Then,

$$Y(s) = \frac{G_p K_c}{1 + G_p K_c} Y_{sp}(s) + \frac{G_d}{1 + G_p K_c} d(s)$$

# Effect of Proportional Control (contd..)

Substituting for $G_p$ and $G_d$, we get,

$$\begin{aligned}
Y(s) &= \frac{\dfrac{K_p}{\tau_p s+1}K_c}{1+\dfrac{K_p}{\tau_p s+1}K_c}Y_{sp}(s) + \frac{\dfrac{K_d}{\tau_p s+1}}{1+\dfrac{K_p}{\tau_p s+1}K_c}d(s) \\
&= \frac{K_pK_c}{\tau_p s+1+K_pK_c}Y_{sp}(s) + \frac{K_d}{\tau_p s+1+K_pK_c}d(s)
\end{aligned}$$

i.e.,

$$\boxed{Y(s) = \frac{K_p'}{\tau_p' s+1}Y_{sp}(s) + \frac{K_d'}{\tau_p' s+1}d(s)}$$

where

$$\tau_p' = \frac{\tau_p}{1+K_pK_c} \qquad K_p' = \frac{K_pK_c}{1+K_pK_c} \qquad K_d' = \frac{K_d}{1+K_pK_c}$$

The parameters $K_p'$ and $K_d'$ are known as *closed-loop static gains*.

# Effect of Proportional Control (contd..)

Closed loop response of first order system due to proportional control has the following characteristics:

- It remains first-order with respect to load and setpoint changes.
- The time constant has been reduced (i.e., $\tau_p' < \tau_p$), which means that the closed loop response become faster, than the open loop response, to changes in setpoint or load.
- The static gains have been decreased.

# Effect of Proportional Control (contd..)

## Unit Step Change in Setpoint

$$\text{Setpoint change} \implies \text{servo problem}$$
$$\text{Load change} \implies \text{regulator problem}$$

For the servo problem, $Y_{sp} = 1/s$, and $d(s) = 0$. Hence the response becomes,

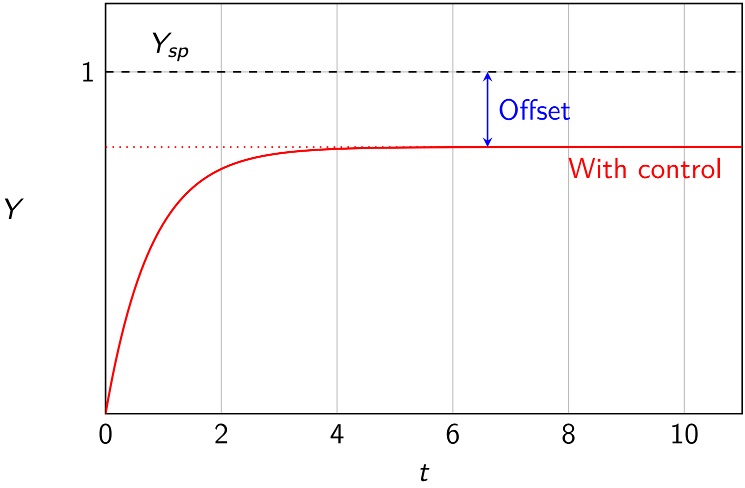
$$Y(s) = \frac{K_p'}{\tau_p' s + 1}\frac{1}{s}$$

Taking inverse Laplace transform, we get

$$Y(t) = K_p'(1 - e^{-t/\tau_p'})$$

# Effect of Proportional Control (contd..)

## Unit Step Change in Setpoint




Offset is the characteristic effect of proportional control. Offset can be reduced by increasing the controller gain.

For unit step change in load, $d(s) = 1/s$. It can be verified, that, here too the controlled system has offset.

Closed loop response of a second order system with proportional control has the following characteristics:

- It remains second order.
- Damping factor decreases. $\Longrightarrow$ Originally overdamped process may become underdamped.
- Offset $\neq 0$.

# Effect of Integral Control

Closed loop system with $G_m = G_f = 1$ leads to,

$$Y(s) = \frac{G_p G_c}{1 + G_p G_c} Y_{sp}(s) + \frac{G_d}{1 + G_p G_c} d(s)$$

Let us study the effect of setpoint change, i.e., servo problem.
Here, $d(s) = 0$.
Let us consider a first order system, controlled by integral only controller. For the integral control action,

$$C(t) = K_c \frac{1}{\tau_I} \int_0^t \epsilon(t) dt$$

Taking Laplace transform,

$$C(s) = K_c \frac{1}{\tau_I s} \epsilon(s) = \qquad \Longrightarrow \qquad \frac{C(s)}{\epsilon(s)} = G_c$$

$$G_c = K_c \frac{1}{\tau_I s}$$

where $K_c$ is controller gain; and, $\tau_I$ is integral time.

The closed loop response for the setpoint changes is given by

$$Y(s) = \frac{G_p G_c}{1 + G_p G_c} Y_{sp}(s) = \frac{\left(\dfrac{K_p}{\tau_p s + 1}\right)\left(\dfrac{K_c}{\tau_I s}\right)}{1 + \left(\dfrac{K_p}{\tau_p s + 1}\right)\left(\dfrac{K_c}{\tau_I s}\right)} Y_{sp}(s)$$

$$= \frac{K_p K_c}{\tau_p \tau_I s^2 + \tau_I s + K_p K_c} Y_{sp}(s)$$

$$\Longrightarrow \quad Y(s) = \frac{1}{\tau^2 s^2 + 2\zeta\tau s + 1} Y_{sp}(s)$$

where

$$\tau^2 = \frac{\tau_p \tau_I}{K_p K_c} \quad \Longrightarrow \quad \tau = \sqrt{\frac{\tau_p \tau_I}{K_p K_c}}$$

# Effect of Integral Control (contd..)

$$2\zeta\tau = \frac{\tau_I}{K_p K_c} \quad \Longrightarrow \quad \zeta\sqrt{\frac{\tau_p \tau_I}{K_p K_c}} = \frac{1}{2}\frac{\tau_I}{K_p K_c}$$

$$\Longrightarrow \quad \zeta = \frac{1}{2}\sqrt{\frac{\tau_I}{\tau_p K_p K_c}}$$

With integral controller, order of the system is increased.

Ultimate value for unit step change in point: $Y_{sp}(s) = 1/s$.

$$\lim_{t\to\infty} Y(t) = \lim s \to 0 sY(s) = \lim_{s\to 0} \frac{1}{\tau^2 s^2 + 2\zeta\tau s + 1} = 1$$

Therefore,

$$\text{offset} = Y_{sp} - Y_{t\to\infty} = 1 - 1 = 0$$

Integral action eliminates offset.

# Effect of Derivative Control Action

$$C(t) = K_c \tau_D \frac{d\epsilon(t)}{dt}$$

$$G_c = K_c \tau_D s$$

where $\tau_D$ = derivative time.
Closed loop response of a first order system with derivative control is given by

$$Y(s) = \frac{\dfrac{K_p}{\tau_p s + 1} K_c \tau_D s}{1 + \dfrac{K_p}{\tau_p s + 1} K_c \tau_D s} Y_{sp}(s)$$

$$= \frac{K_p K_c \tau_D s}{(\tau_p + K_p K_c \tau_D)s + 1} Y_{sp}(s)$$

- The derivative control does not change the order of the response.
- The effective time constant of the closed-loop response is $(\tau_p + K_p K_c \tau_D)$, i.e., larger than $\tau_p$. This means that the response of the controlled process is slower than that of the original first-order process.

# Effect of Derivative Control Action (contd..)

- For constant nonzero error, derivative mode does not give any output.
- This mode is never used alone. It is always used in combination with proportional or proportional-plus-integral control action.
- Derivative mode should not be used on noisy loops (e.g.: flow).