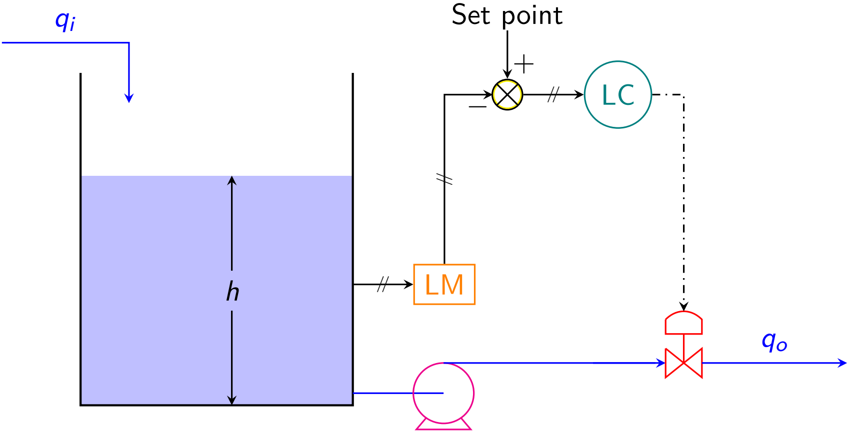
Control of Liquid Level



Tank dynamics:

$$q_i - q_o = A\frac{dh}{dt}$$

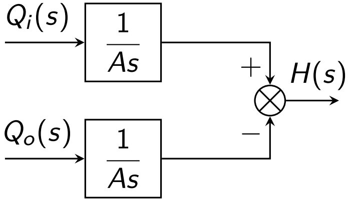
In terms of deviation variables,

$$Q_i - Q_o = A\frac{dH}{dt}$$

Taking Laplace transforms, and rearranging,

$$H(s) = \frac{1}{As}(Q_i(s) - Q_o(s)) = G_d Q_i(s) - G_p Q_o(s)$$

Block Diagram Representation:



Outflow from the pump is regulated by a control valve (the final control element), dynamics of which is given by

$$Q_o(s) = \frac{K_v}{\tau_v s + 1} C(s) \qquad \Longrightarrow \qquad Q_o(s) = G_f C(s)$$

Signal to the control valve is from a controller (typically a PID controller), and the dynamics of which is given by

$$C(s) = G_c \epsilon(s)$$

For the PID controller,

$$G_c = K_c \left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Signal to the controller is the error amount. i.e.,

$$\epsilon(s) = H_{sp}(s) - H_m(s)$$

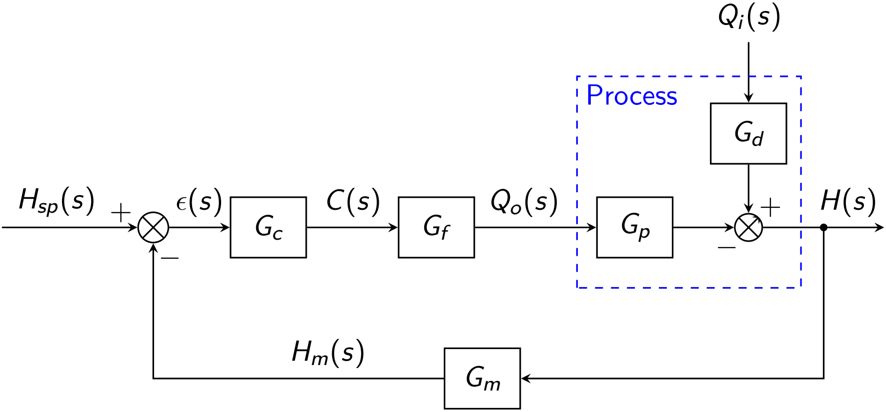
The measurement device for level may be differential pressure cell (DPC), which has second order dynamics, as given by

$$\frac{H_m(s)}{H(s)} = \frac{K_m}{\tau^2 s^2 + 2\tau\zeta s + 1} \qquad \Longrightarrow \qquad H_m(s) = G_m H(s)$$

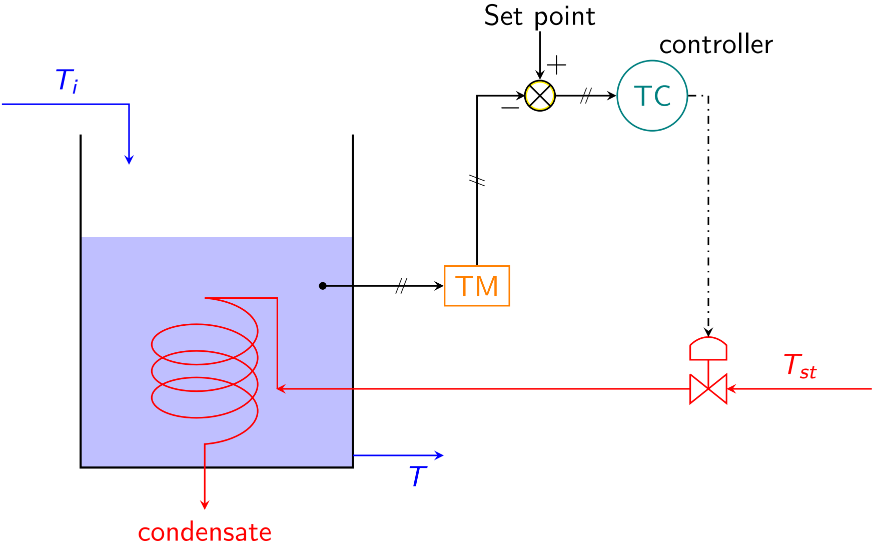
Control of Liquid Level (contd..)

Hence, for the closed loop

$$
\begin{aligned}
H(s) &= G_d Q_i(s) - G_p Q_o(s) \\
&= G_d Q_i(s) - G_p G_f C(s) \\
&= G_d Q_i(s) - G_p G_f G_c \epsilon(s) \\
&= G_d Q_i(s) - G_p G_f G_c \left[H_{sp}(s) - H_m(s)\right] \\
&= G_d Q_i(s) - G_p G_f G_c \left[H_{sp}(s) - G_m H(s)\right] \\
(1 - G_p G_f G_c G_m) H(s) &= G_d Q_i(s) - (G_p G_f G_c) H_{sp}(s) \\
H(s) &= \frac{G_d}{1 - G_p G_f G_c G_m} Q_i(s) - \frac{G_p G_f G_c}{1 - G_p G_f G_c G_m} H_{sp}(s)
\end{aligned}
$$


$Q_i(s)$
Process
G_d
$H_{sp}(s)$
+
$\epsilon(s)$
G_c
$C(s)$
G_f
$Q_o(s)$
G_p
+
$-$
$H(s)$
$-$
$H_m(s)$
G_m

Temperature Control of Tank Heater Heated by Steam



Response of Stirred Tank Heater

Energy balance: (constant density, constant volume, constant flow system)

$$FC_P\rho(T_i - T) + Q = V\rho C_P\frac{dT}{dt}$$

Heat input by steam is given by

$$Q = UA(T_{st} - T)$$

Therefore, energy balance equation becomes,

$$V\frac{dT}{dt} + \left(F + \frac{UA}{\rho C_P}\right)T = FT_i + \frac{UA}{\rho C_P}T_{st}$$

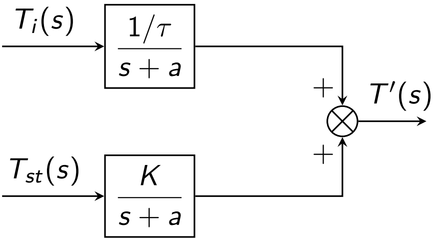
Substituting, $\frac{F}{V} = \frac{1}{\tau}$, and $K = \frac{UA}{V\rho C_P}$, and $a = \frac{1}{\tau} + K$, we get

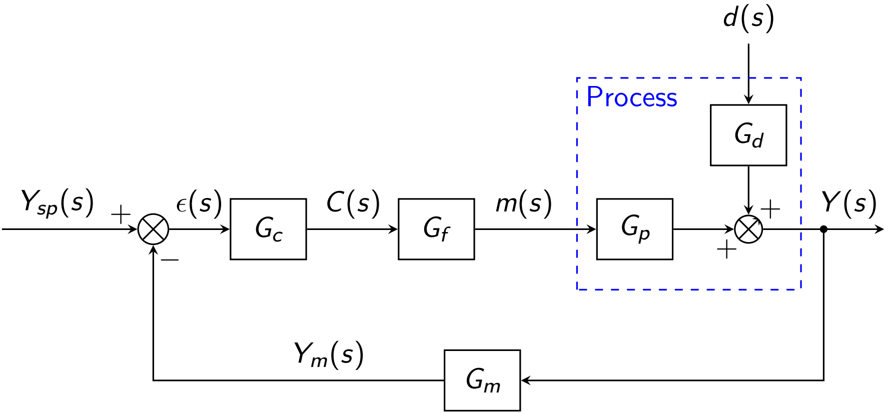
$$\frac{dT}{dt} + aT = \frac{1}{\tau}T_i + KT_{st}$$

Using deviation variables and taking Laplace transform, we get

$$(s+a)T'(s) = \frac{1}{\tau}T_i'(s) + KT_{st}'(s)$$
$$T'(s) = \frac{1/\tau}{s+a}T_i'(s) + \frac{K}{s+a}T_{st}'(s)$$

Block Diagram Representation:

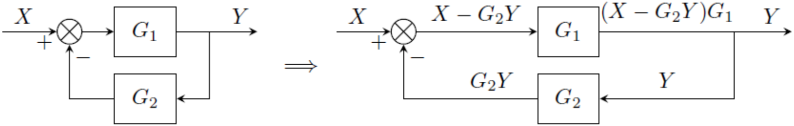




m - manipulated variable; d - disturbance.

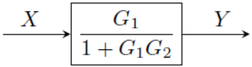
$$Y(s) = \frac{G_p G_f G_c}{1 + G_p G_f G_c G_m} Y_{sp}(s) + \frac{G_d}{1 + G_p G_f G_c G_m} d(s)$$

Block Diagram Reduction



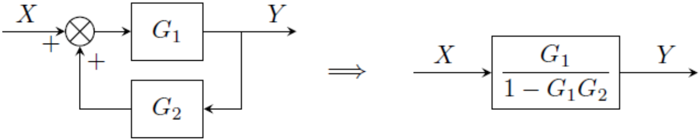
$$\Longrightarrow \quad (X - G_2Y)G_1 = Y \quad \Longrightarrow \quad XG_1 = Y + G_1G_2Y \quad \Longrightarrow \quad X\left(\frac{G_1}{1 + G_1G_2}\right) = Y$$

$\Longrightarrow$



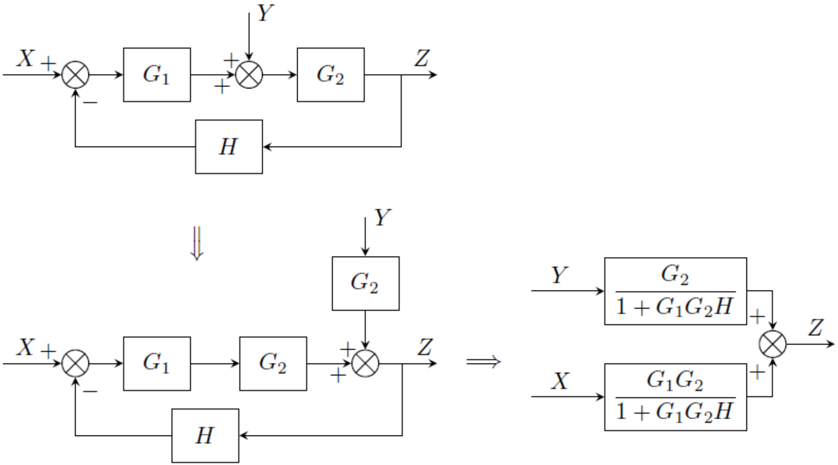
(a) Negative Feedback Loop

Block Diagram Reduction (contd..)



(b) Positive Feedback Loop

Block Diagram Reduction (contd..)



(c) Feedback Loop with Load Change