CH2404 Process Economics

Unit – II

www.msubbu.in

Time Value of Money

Dr. M. Subramanian

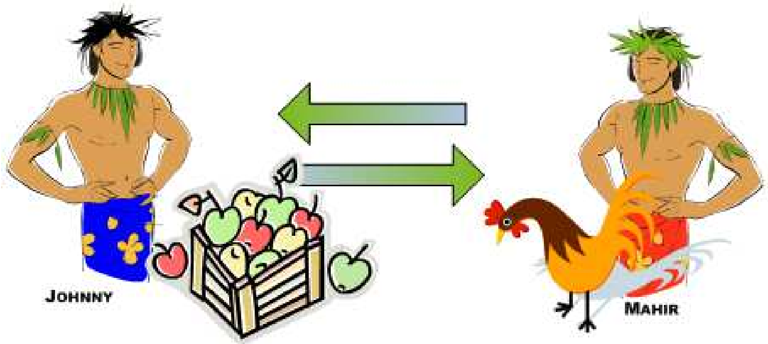
Associate Professor
Department of Chemical Engineering
Sri Sivasubramaniya Nadar College of Engineering
Kalavakkam – 603 110, Kanchipuram (Dist)
Tamil Nadu, India
msubbu.in[AT]gmail.com

Contents

- Time Value of Money
- Compound Interest
- Future Value
- Present Value
- Frequency of Compounding
- Annuities

Money

- In early primitive civilizations, trade and business were based on a direct exchange of goods (Barter system).



- As civilizations developed, money was introduced to facilitate the exchange of goods and services.

- Money has value only when it is spent. It would be of little use to an individual on a desert island.

- Money is only received on the understanding that it can be passed on again in exchange for something else.

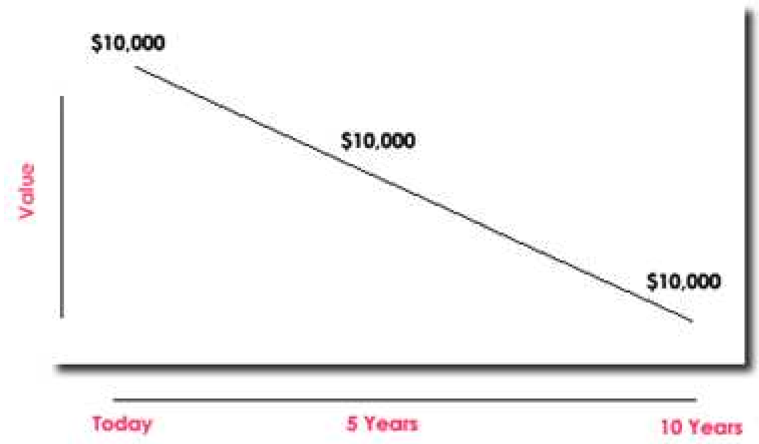
Time value of Money

Which would you rather have ??

$1,000 today or $1,000 in 5 years?

Obviously, $1,000 today.

Money received sooner rather than later allows one to use the funds for investment or consumption purposes. This concept is referred to as the *TIME VALUE OF MONEY!!*



Remember, one CANNOT compare numbers in *different time periods* without first adjusting them using an interest rate.

Interest

- Interest is a charge for borrowing money, usually stated as a percentage of the amount borrowed over a specific period of time.

- A large part of business activity is based on the fact that money can be borrowed or loaned. When money is loaned there is always a risk that it may not be returned. Interest is the inducement offered to make the risk acceptable.

- “When money is lent on a contract to receive not only the principal sum again, but also an increase by way of compensation for the use, the increase is called **interest** by those who think it lawful, and **usury** by those who do not." (*Blackstone's Commentaries on the Laws of England*, pp.1336, 1769).

- In economics, interest is considered the price of credit.

- Interest is often compounded, which means that interest is earned on prior interest in addition to the principal. The total amount of debt grows exponentially.

Interest Rate

An **interest rate** is the cost stated as a percent of the amount borrowed per period of time, usually one year. The prevailing market rate is composed of:

- The *Real Rate of Interest* that compensates lenders for postponing their own spending during the term of the loan.
- An *Inflation Premium* to offset the possibility that inflation may erode the value of the money during the term of the loan. A unit of money (rupee, dollar, etc) will purchase progressively fewer goods and services during a period of inflation, so the lender must increase the interest rate to compensate for that loss..
- Various *Risk Premiums* to compensate the lender for risky loans such as those that are unsecured, made to borrowers with questionable credit ratings, or illiquid loans that the lender may not be able to readily resell.

Simple Interest

$$F = P + I = P + Pin = P(1 + in)$$

- The interest is charged on the original loan and not on the unpaid balance.
- Simple interest is paid at the end of each time interval.
- Although the simple interest concept still exists, it is seldom used.

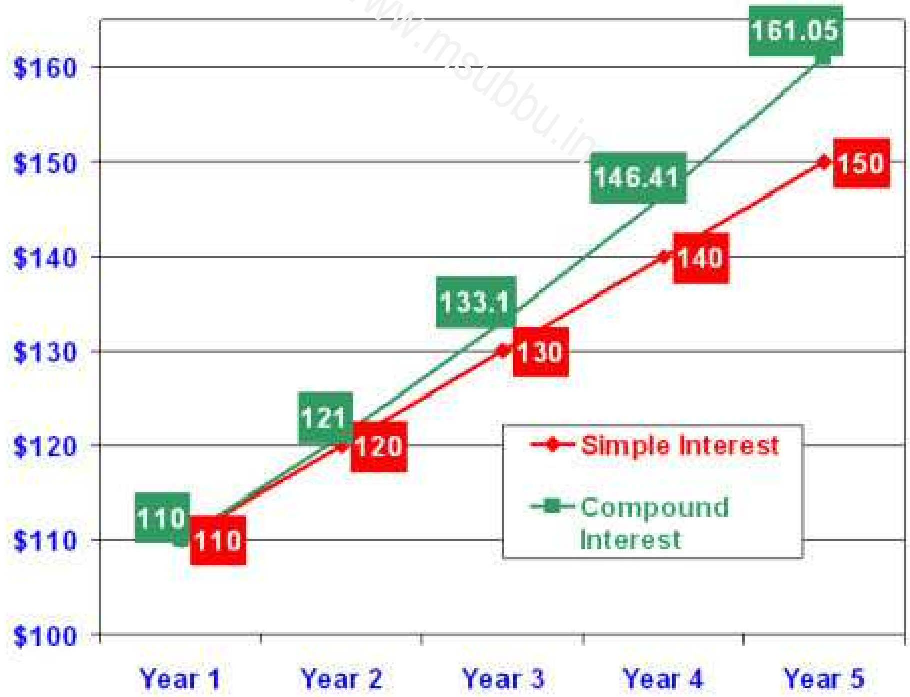
Compound Interest

- When interest is paid on not only the principal amount invested, but also on any previous interest earned, this is called compound interest.

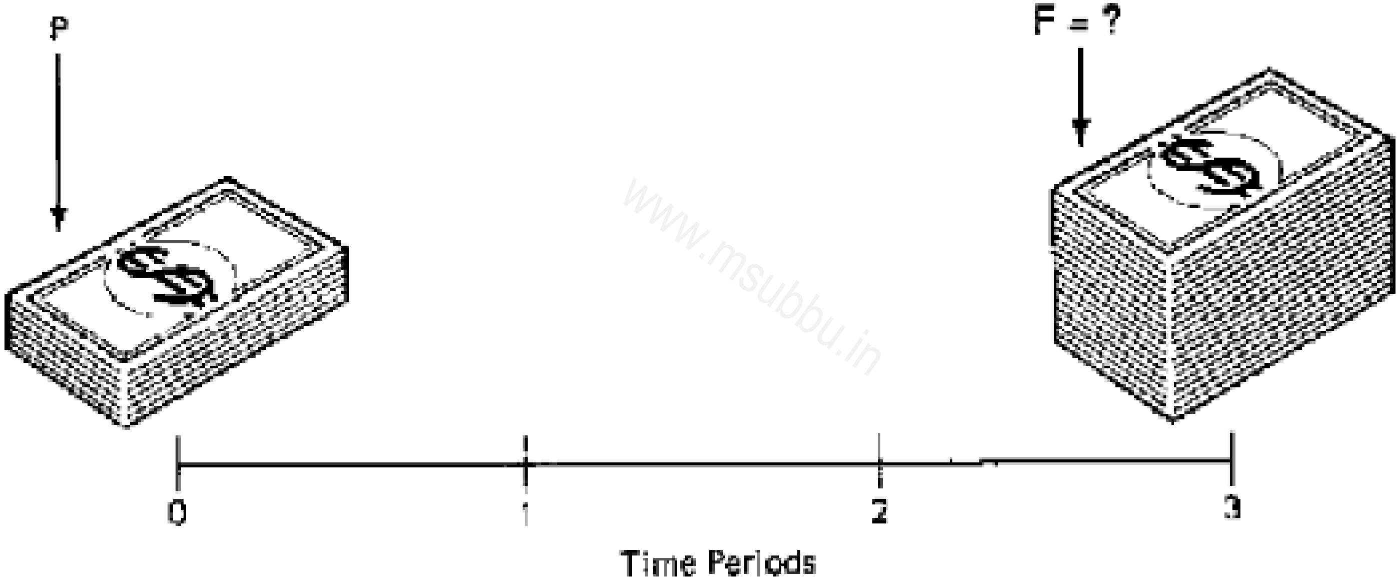
- Since interest has a time value, often the lender will invest this interest and earn more additional interest. It is assumed that the interest is not withdrawn but is added to the principal and then in the next period interest is calculated based upon the principal plus the interest in the preceding period.

$$F_1 = P + Pi = P(1 + i)$$
$$F_2 = P(1 + i) + P(1 + i)i = P(1 + i)^2$$
$$F_3 = P(1 + i)^2 + P(1 + i)^2 i = P(1 + i)^3$$
$$\vdots$$
$$F_n = P(1 + i)^n$$

Year	Beginning Amount	Simple Interest	Interest on Interest	Total Interest Earned	Total Ending Amount
1	$100	$10	$0	$10	$110
2	$110	$10	$1	$11	$121
3	$121	$10	$2.1	$12.10	$133.10
4	$133.10	$10	$3.31	$13.31	$146.41
5	$146.41	$10	$4.64	$14.64	$161.05

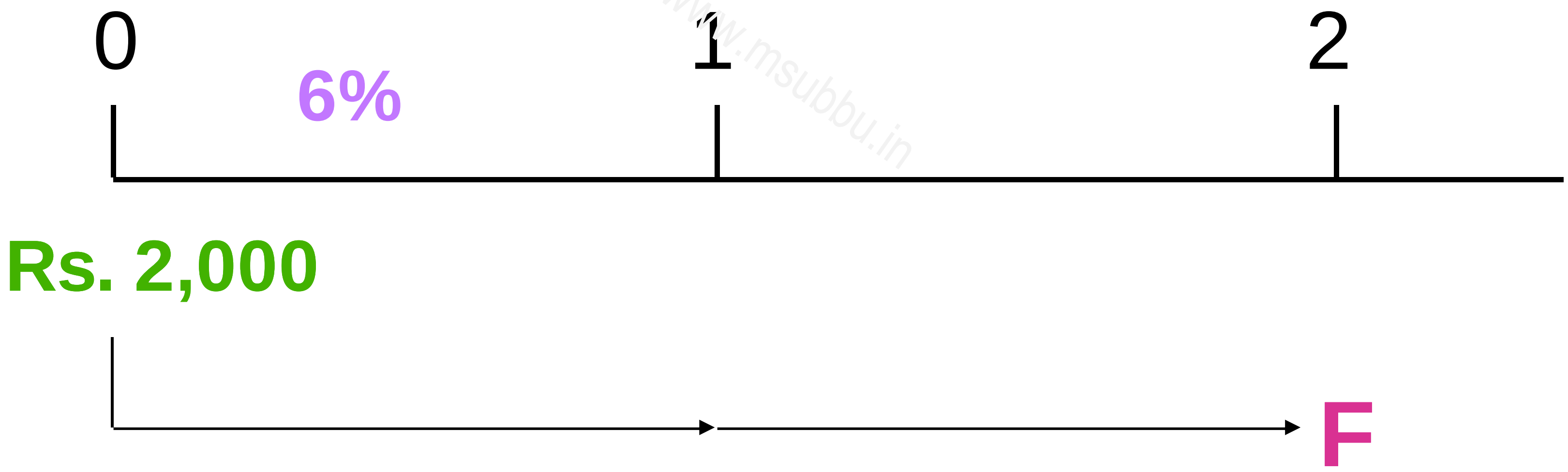


Future value



Future value - example

If you invested Rs. 2,000 today in an account that pays 6% interest, with interest compounded annually, how much will be in the account at the end of two years if there are no withdrawals?

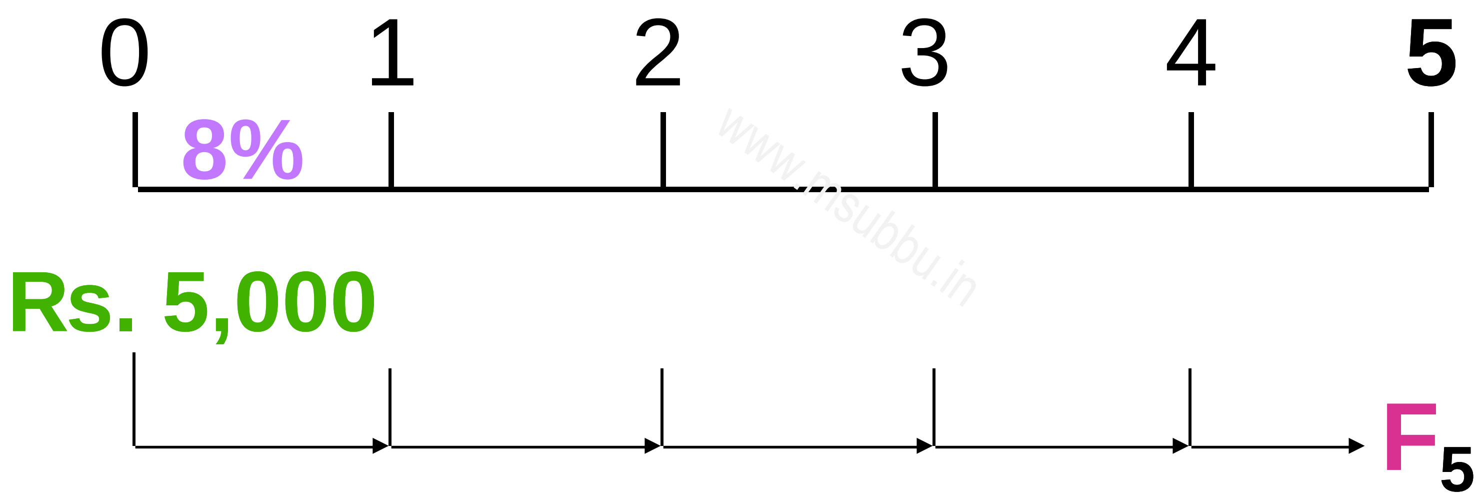


$F_2 = P\,(1+i)^n$ $\qquad = \text{Rs. } 2{,}000\,(1.06)^2$

$= \text{Rs. } 2{,}247.20$

Future value – example

John wants to know how large his Rs. 5,000 deposit will become at an annual compound interest rate of 8% at the end of 5 years.



$$F_n = P\,(1+i)^n$$

$$F_5 = \text{Rs. } 5{,}000\,(1+0.08)^5$$

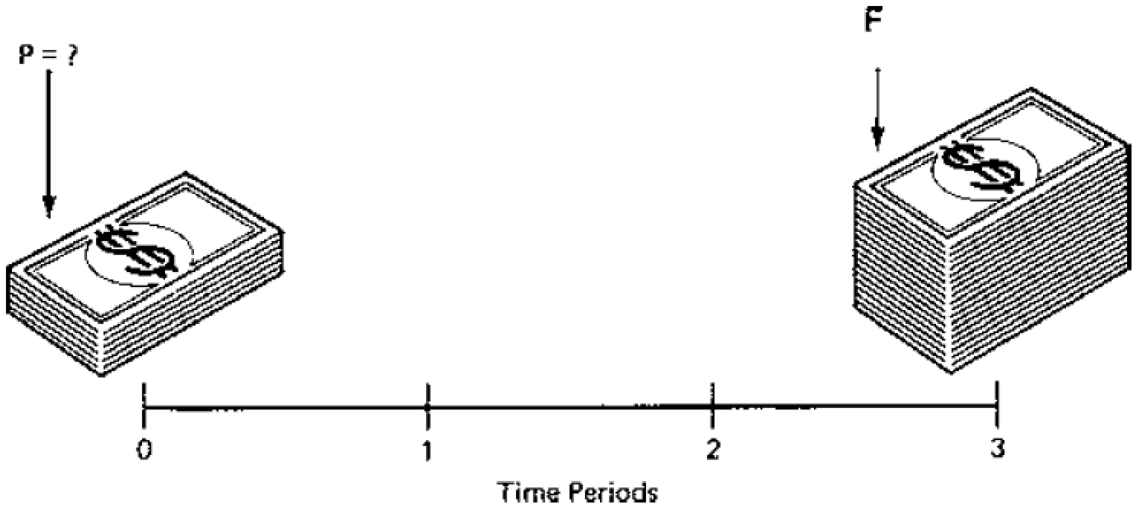
$$= \text{Rs. } 7{,}346.64$$

Present Value

- Since $F = P(1 + i)^n$

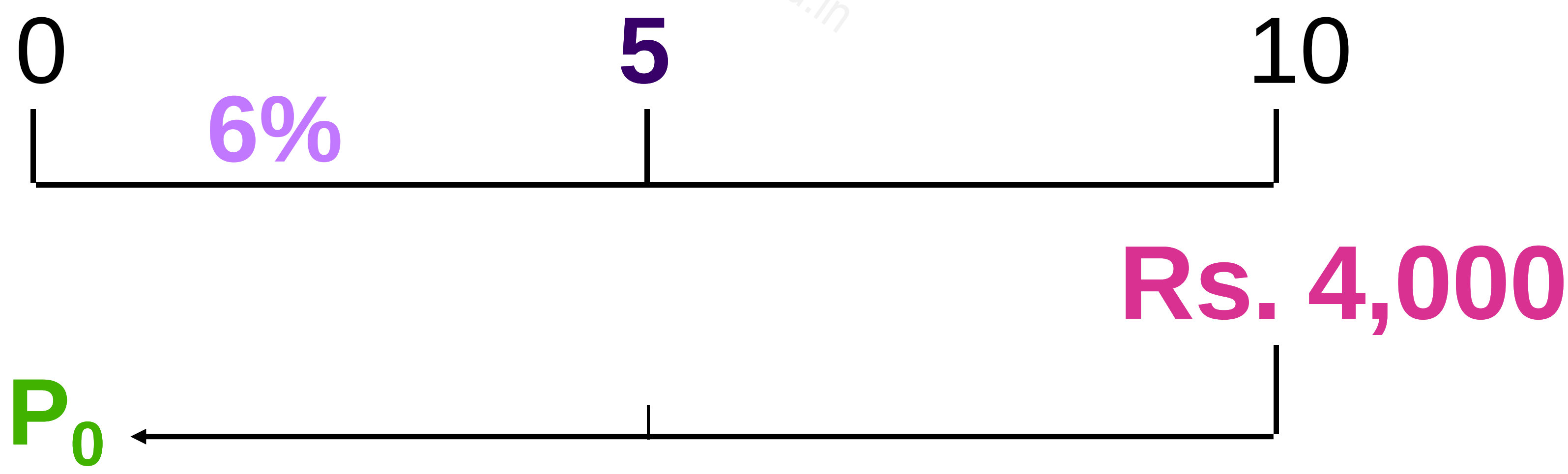
  $$P = F / (1+i)^n$$

- Discounting is the process of translating a future value or a set of future cash flows into a present value.



Present value - example

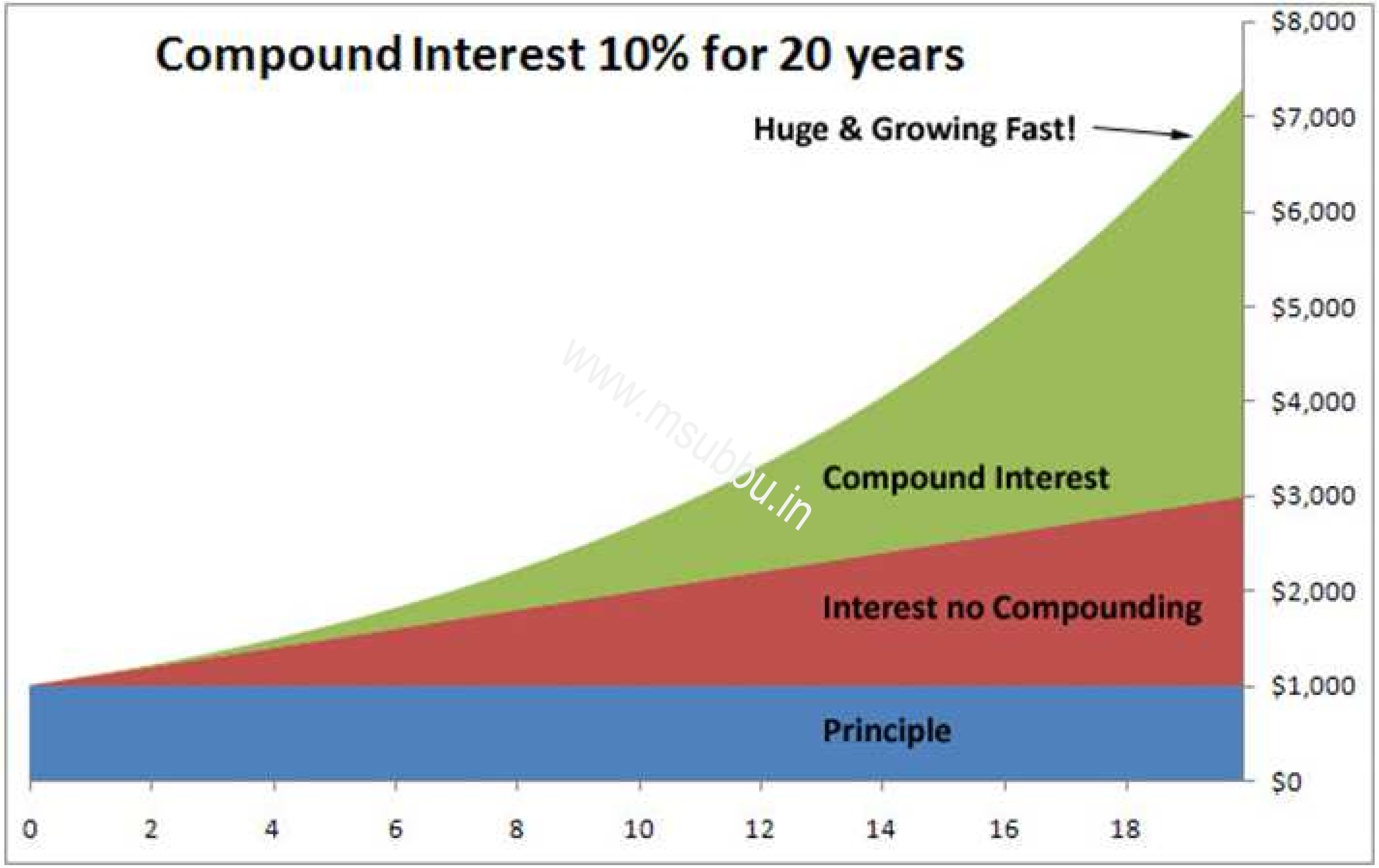
$$P_0 = F / (1+i)^2 \qquad = \text{Rs. } 4{,}000 / (1.06)^{10}$$
$$= \text{Rs. } 2{,}233.58$$



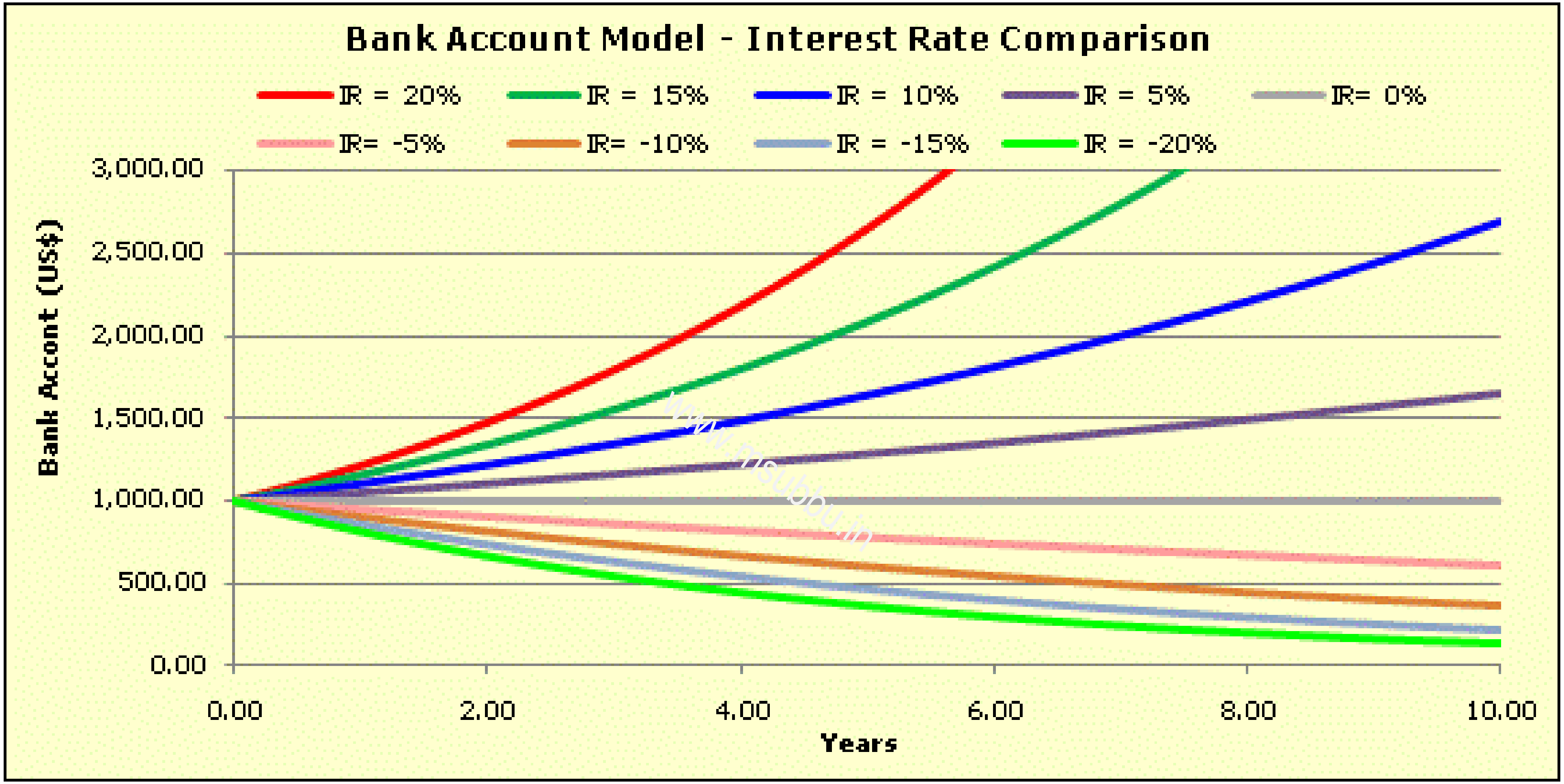
Power of Compounding

P = Rs. 100/-

At end of Year	5%	10%	15%	20%
1	105	110	115	120
5	128	161	201	249
10	163	259	405	619
15	208	418	814	1541
25	**339**	**1,083**	**3,292**	**9,540**



10% - compounded quarterly



Bank Account Behavior at positive and negative interest rates with **quarterly** interest payments

Negative interest rate is due to inflation

Frequency of Compounding

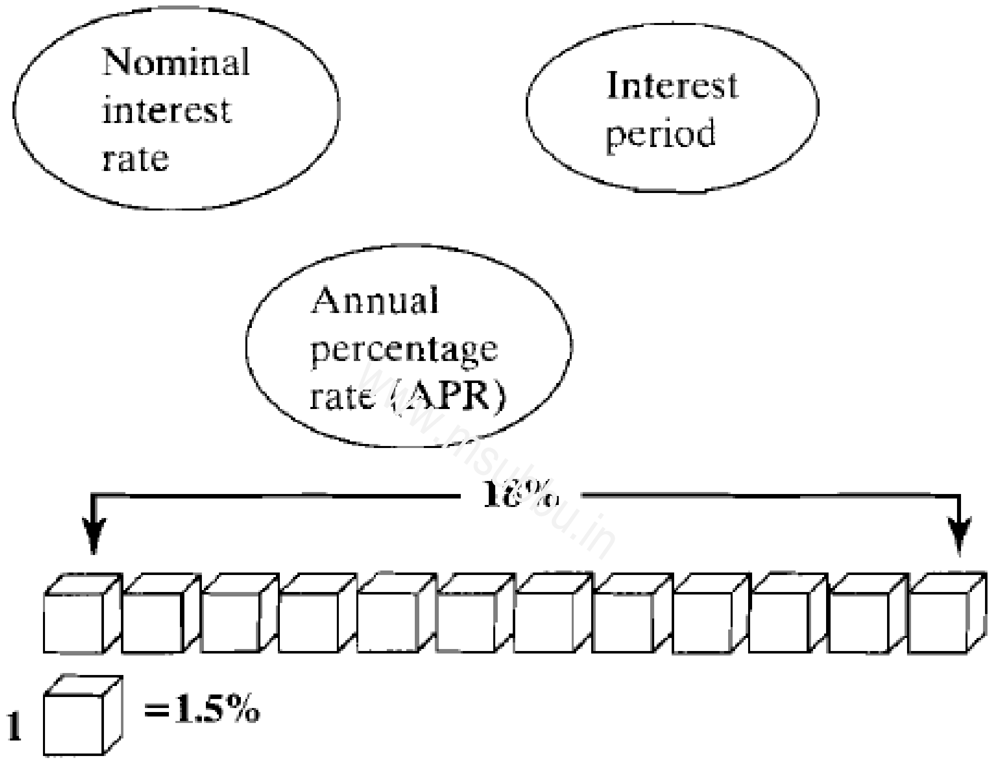
- Although the interest may be stated as a yearly rate, the compounding periods can be yearly, semiannually, quarterly, or even continuously.

- General Formula:

$$F_n = P_0(1 + [i/m])^{mn}$$

n: Number of Years
m: Compounding Periods per Year
i: Annual Interest Rate
F_n : FV at the end of Year n
P_0: PV of the Cash Flow today

18% Compounded Monthly



"18% compounded monthly."

This statement means simply that each month the bank will charge 1.5% interest (12 months per year × 1.5% per month = 18% per year) on the unpaid balance.

F at the end of n years with m compunding periods per year can be calculated from the formula

$$\boxed{F_n = P\left(1+\frac{i}{m}\right)^{mn}}$$

The above formular can be derived in the following way:

$$\begin{aligned}
F \text{ at the end of 1 compunding period} &= P + P\times\frac{i}{m} = P\left(1+\frac{i}{m}\right)\\
F \text{ at the end of 2 compunding periods} &= P\left(1+\frac{i}{m}\right) + P\left(1+\frac{i}{m}\right)\times\left(\frac{i}{m}\right)\\
&= P\left(1+\frac{i}{m}\right)\left(1+\frac{i}{m}\right) = P\left(1+\frac{i}{m}\right)^2\\
F \text{ at the end of 3 compunding periods} &= P\left(1+\frac{i}{m}\right)^2 + P\left(1+\frac{i}{m}\right)^2\times\left(\frac{i}{m}\right)\\
&= P\left(1+\frac{i}{m}\right)^3\\
F \text{ at the end of 4 compunding periods} &= P\left(1+\frac{i}{m}\right)^3 + P\left(1+\frac{i}{m}\right)^3\times\left(\frac{i}{m}\right)\\
&= P\left(1+\frac{i}{m}\right)^4
\end{aligned}$$

If the compunding period is quarterly, then number of compounding periods per year (m) is 4.

For compounding quarterly, m is 4; and at the end of 2 compounding periods, $n = 2/4$. Hence $mn = 4\times 2/4 = 2$. Similary at the end of 4 compounding periods, $n = 4/4 = 1$, which gives $mn = 4\times 1 = 4$.

Power of Frequency of Compounding

- The power of compounding can have an astonishing effect on the accumulation of wealth. This table shows the results of making a one-time investment of Rs. 10,000 for 30 years using 12% simple interest, and 12% interest compounded yearly and quarterly.

Type of Interest	**Principal Plus Interest Earned (Rs.)**
Simple	46,000
Compounded Yearly	2,99,599
Compounded Quarterly	3,47,110

Calculations for the previous table

- Simple interest:

  $F = P(1 + ni) = 10{,}000(1 + 30 \times 0.12) = 46{,}000$

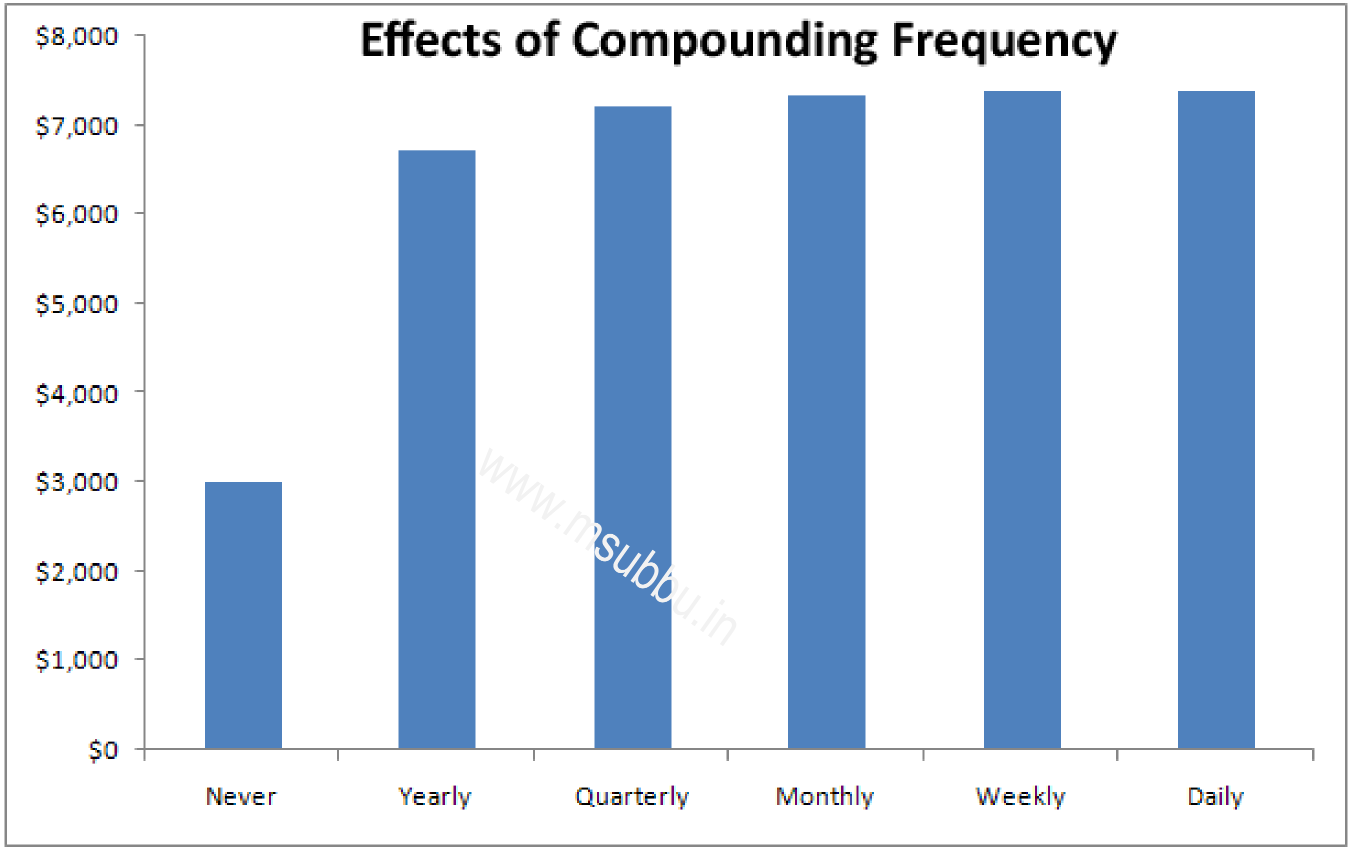
- Compounded yearly

  $F = P(1 + i)^n = 10{,}000(1 + 0.12)^{30} = 2{,}99{,}599$

- Compounded quarterly

  Here, $n = 30 \times 4 = 120$; and $i = 0.12/4 = 0.03$

  $F = P(1 + i)^n = 10{,}000(1 + 0.03)^{120} = 3{,}47{,}110$



$1000 earning a 10% return with no compounding vs compounding yearly / quarterly / monthly / weekly / daily over 20 years:

More frequent is always better but the effect diminishes rapidly

Deposit: $1000; Interest rate: 12% per year

Compounding Frequency	Period	Rate per Compounding Period, *i*	Number of Periods in 5 Years, *N*	*FV* at the End of Five Years
Annual	one year	12%	5	$1,762.34
Semiannual	six months	6%	10	1,790.85
Quarterly	three months	3%	20	1,806.11
Monthly	one month	1%	60	1,816.70

Frequency of Compounding – Example 1

- Suppose you deposit \$1,000 in an account that pays 12% interest, compounded quarterly. How much will be in the account after eight years if there are no withdrawals?

P = \$1,000

i = 12%/4 = 3% per quarter

n = 8 x 4 = 32 quarters

$$F = P(1 + i)^n$$

$$= 1{,}000(1.03)^{32}$$

$$= 2{,}575.10$$

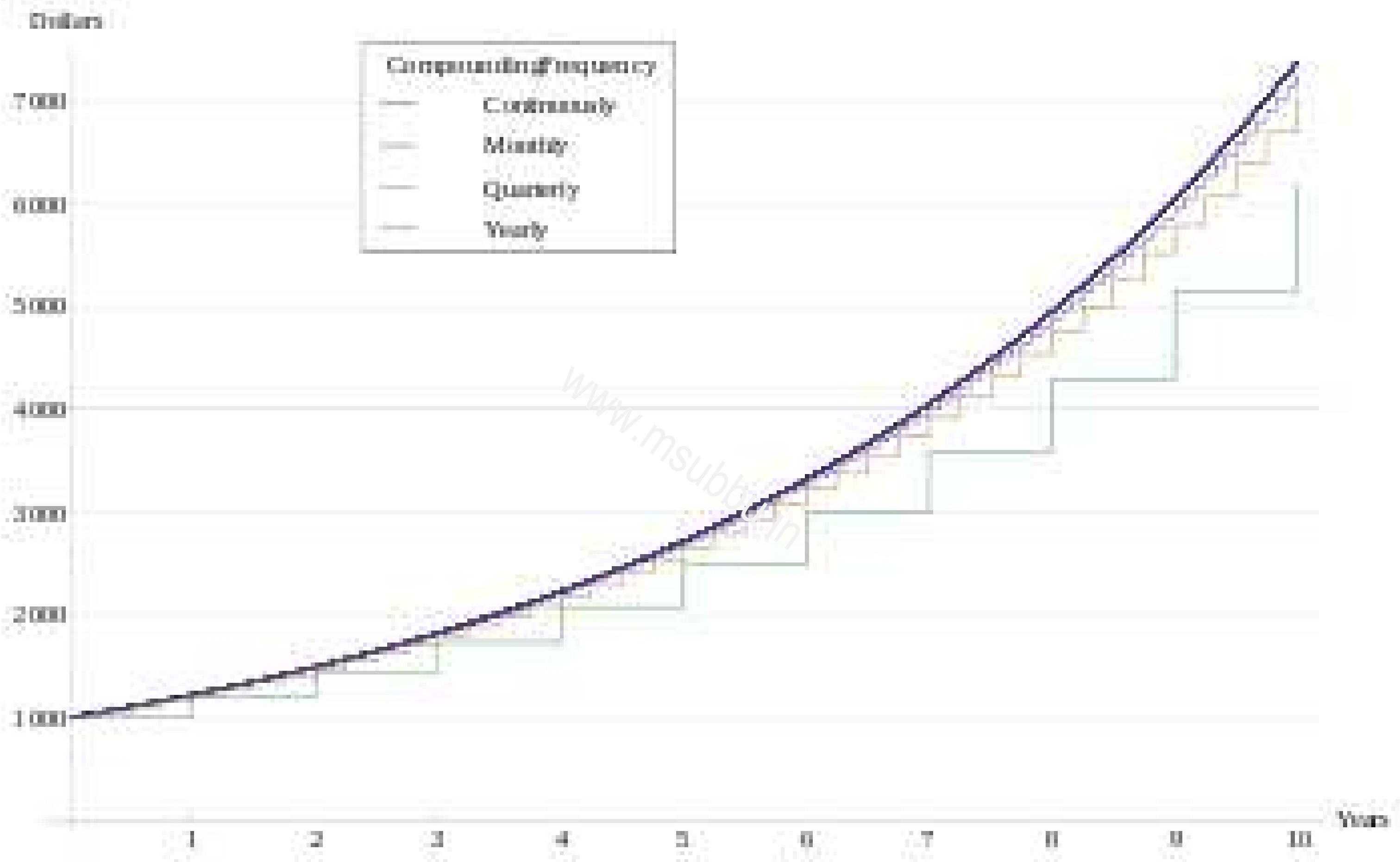
Frequency of Compounding – Example 2

Bank A offers deposit at the rate of **8%** compounded yearly, Bank B offers deposit at the same rate of 8% compounded half yearly Bank C 8% compounded quarterly. Which one would you choose and why?

On Principal of 1 lakh, Maturity value for different compounding period is given below.

Interest would be = Maturity value – Principal

Period of Holding	**Bank A (compounded yearly)**	**Bank B (compounded half yearly)**	**Bank C (compounded quarterly)**
1 year	$1,00,000(1+0.08)$ = 1,08,000	$1,00,000(1+0.08/2)^{2x1}$ = **1,08,160**	$1,00,000(1+0.08/4)^{4x1}$ = **1,08,243**
2 year	$1,00,000(1+0.08)^{2}$ = 1,16,640	$1,00,000(1+0.08/2)^{2x2}$ = **1,16,986**	$1,00,000(1+0.08/4)^{4x2}$ = **1,17,166**
5 year	$1,00,000(1+0.08)^{5}$ = 1,46,933	$1,00,000(1+0.08/2)^{2x5}$ = **1,48,024**	$1,00,000(1+0.08/4)^{4x5}$ = **1,48,595**



The effect of earning 20% annual interest on an initial $1,000 investment at various compounding frequencies

Effective Annual Interest Rate

- If compounding is done not on yearly but on any other terms such as monthly, quarterly, etc., the effective annual interest rate (i_{eff}) is calculated as:

$$i_{\text{eff}} = (1 + i/m)^m - 1$$

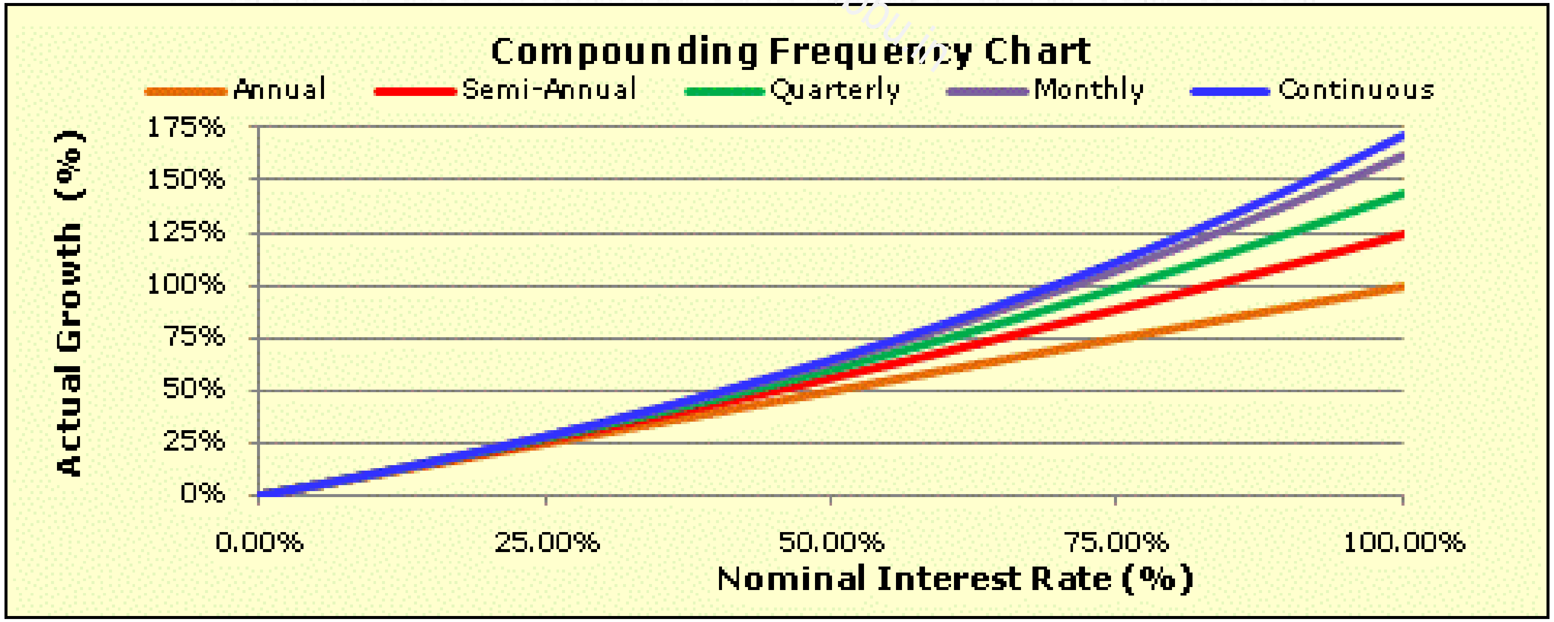
  where i is the nominal annual interest rate; and m is the number of terms per year

- If compounding is done continuously (as, m → ∞), then:

$$i_{\text{eff}} = \exp(i) - 1$$

Nominal Rate	Semi-Annual	Quarterly	Monthly	Daily	Continuous
1%	1.002%	1.004%	1.005%	1.005%	1.005%
5%	5.062%	5.095%	5.116%	5.127%	5.127%
10%	10.250%	10.381%	10.471%	10.516%	10.517%
15%	15.563%	15.865%	16.075%	16.180%	16.183%
20%	21.000%	21.551%	21.939%	22.134%	22.140%
30%	32.250%	33.547%	34.489%	34.969%	34.986%
40%	44.000%	46.410%	48.213%	49.150%	49.182%
50%	56.250%	60.181%	63.209%	64.816%	64.872%

Effective Annual Rate Based on Frequency of Compounding



Annual	Semi - Annual	Quarterly	Monthly	Daily	Continuous
0%	0.00%	0.00%	0.00%	0.00%	0.00%
5%	5.06%	5.09%	5.12%	5.13%	5.13%
10%	10.25%	10.38%	10.47%	10.52%	10.52%
15%	15.56%	15.87%	16.08%	16.18%	16.18%
20%	21.00%	21.55%	21.94%	22.13%	22.14%
25%	26.56%	27.44%	28.07%	28.39%	28.40%
30%	32.25%	33.55%	34.49%	34.97%	34.99%
35%	38.06%	39.87%	41.20%	41.88%	41.91%
40%	44.00%	46.41%	48.21%	49.15%	49.18%
45%	50.06%	53.18%	55.55%	56.79%	56.83%
50%	56.25%	60.18%	63.21%	64.82%	64.87%
55%	62.56%	67.42%	71.22%	73.25%	73.33%
60%	69.00%	74.90%	79.59%	82.12%	82.21%
65%	75.56%	82.63%	88.33%	91.44%	91.55%
70%	82.25%	90.61%	97.46%	101.24%	101.38%
75%	89.06%	98.85%	106.99%	111.54%	111.70%
80%	96.00%	107.36%	116.94%	122.36%	122.55%
85%	103.06%	116.14%	127.33%	133.73%	133.96%
90%	110.25%	125.19%	138.18%	145.69%	145.96%
95%	117.56%	134.52%	149.50%	158.25%	158.57%
100%	125.00%	144.14%	161.30%	171.46%	171.83%

Example Problems

1. What is the effective annual compound interest rate equivalent to a nominal annual compound interest rate of 5% compounded daily?

$$\begin{aligned} i_{\text{eff}} &= (1 + i/m)^m - 1 \\ &= (1 + 0.05/365)^{365} - 1 \\ &= 0.0513 = 5.13\% \end{aligned}$$

2. What is the effective annual compound interest rate equivalent to a nominal compound interest rate of 5% compounded continuously?

$$\begin{aligned} i_{\text{eff}} &= \exp(i) - 1 \\ &= \exp(0.05) - 1 \\ &= 0.0513 = 5.13\% \end{aligned}$$

Monthly Compound Interest

- The annual interest rate equivalent to a compound fractional interest rate of $\boldsymbol{i_m}$ per month is given by:

$$i = (1 + i_m)^{12} - 1$$

- Most people who take out loans based on what superficially appears to be a reasonable interest rate per month are not fully aware of the equivalent high interest rate on an annual basis.

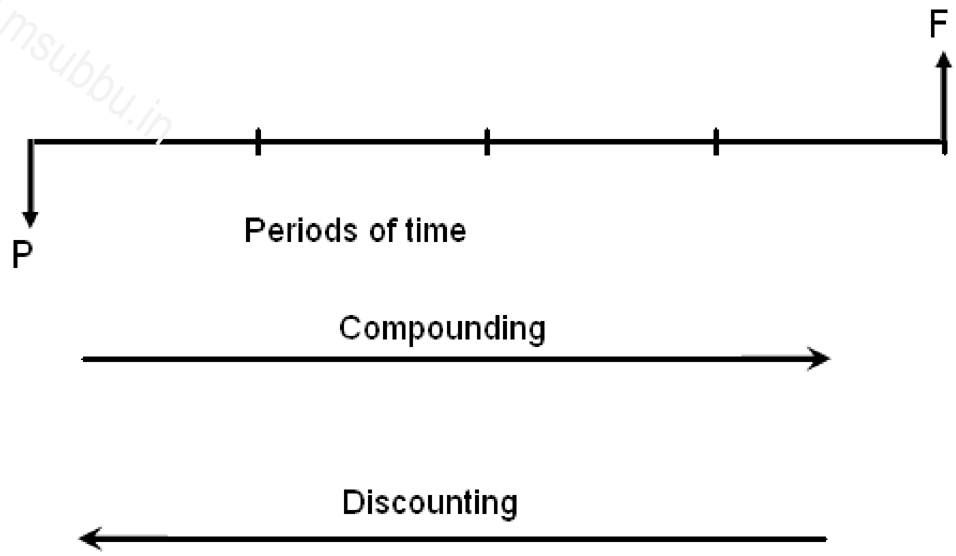
What is the annual interest rate equivalent to a compound interest rate of 3% per month?

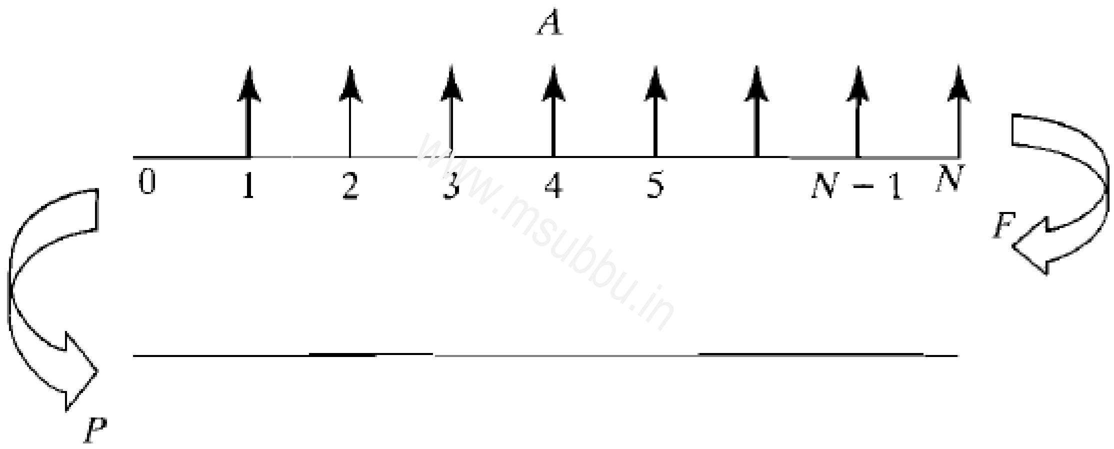
$$\begin{aligned} i &= (1 + i_m)^{12} - 1 \\ &= (1 + 0.03)^{12} - 1 \\ &= 0.426 = 42.6\%/\text{year} \end{aligned}$$

Annuities

- *An Annuity* represents a series of equal payments (or receipts) occurring over a specified number of equidistant periods.

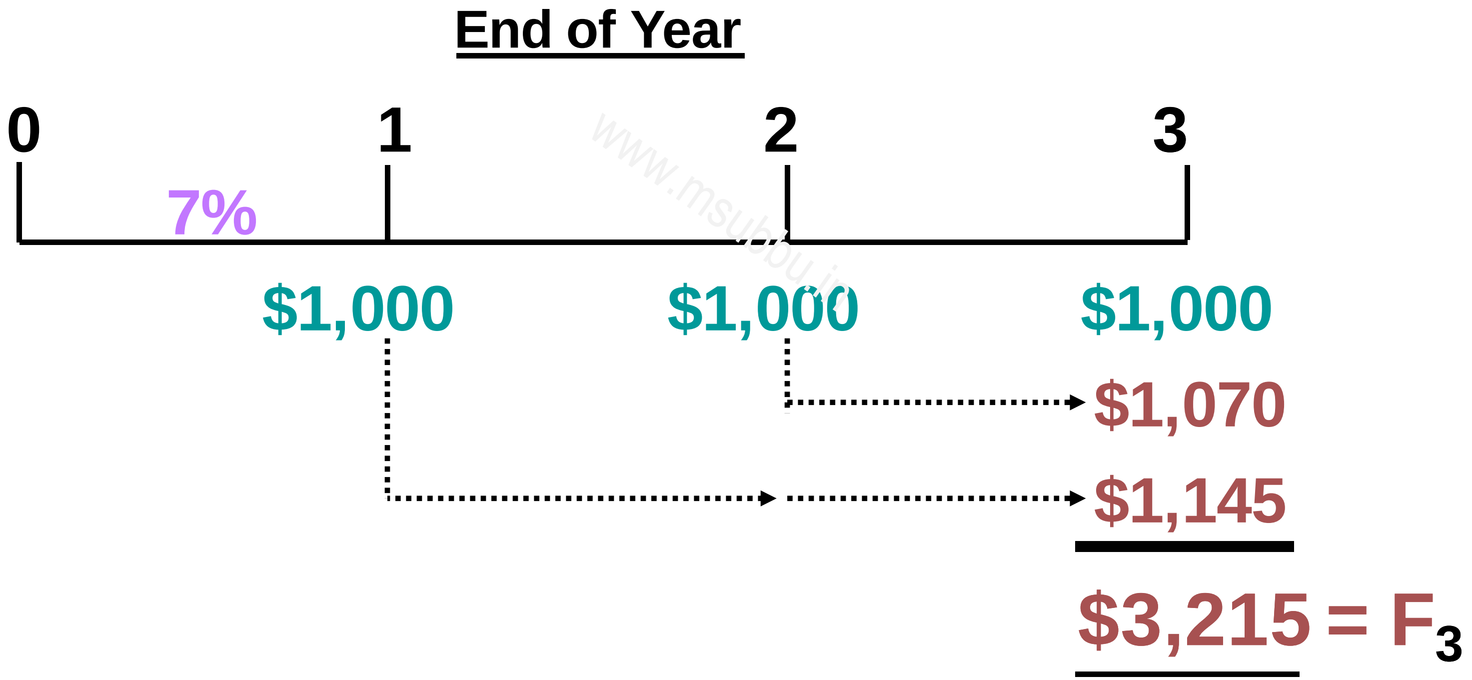
- Examples of Annuities Include:
  - Student Loan Payments
  - Car Loan Payments
  - Insurance Premiums
  - Mortgage Payments
  - Retirement Savings
  - Recurring deposits




A
0
1
2
3
4
5
N − 1
N
F
P

Annuity - Example

If one saves $1,000 a year at the end of every year for three years in an account earning 7% interest, compounded annually, how much will one have at the end of the third year?

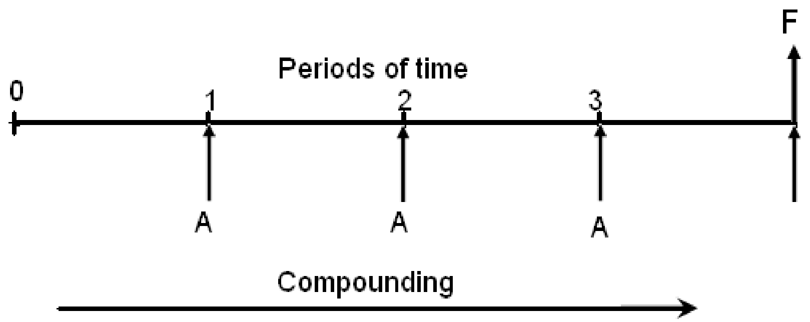


$$F_3 = \$1,000(1.07)^2 + \$1,000(1.07)^1 + \$1,000\,(1.07)^0$$
$$= \$3,215$$

Future worth of annuity

- A series of equal annual payments A invested at a fractional interest rate *i* and made at the end of each year over a period of n years may be used to build up a future sum of money F given by the equation:

$$F = A\left[\frac{(1+i)^n - 1}{(i)}\right]$$



Example problem

- Assuming there is a need for $5000 in 5 years, it was decided to deposit a certain amount of money at the end of every year for 5 years at 4% interest instead of a single sum at time 0. What would the annual amount to be deposited be?

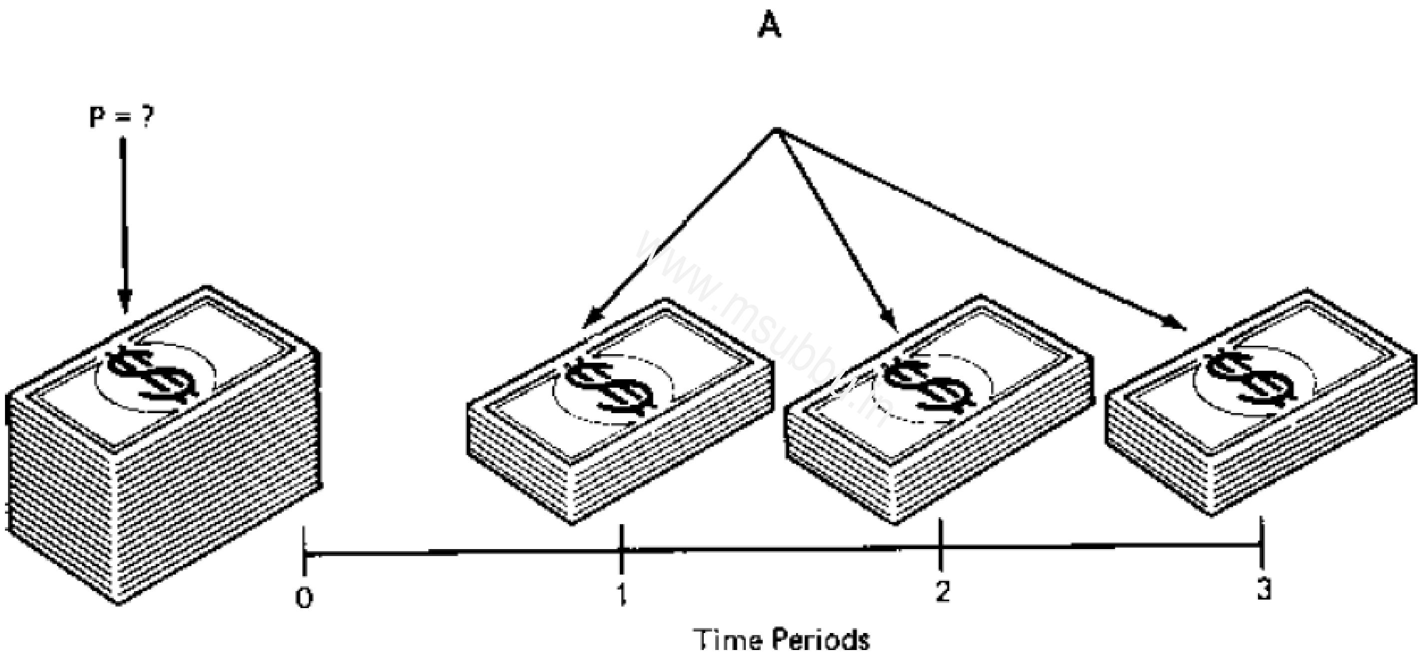
F = $5000; n = 5 years, i = 0.04; and A = ?

$$A = F\left[\frac{(i)}{(1+i)^n - 1}\right]$$

$$A = (\$5000)\left[\frac{(0.04)}{(1.04)^5 - 1}\right] = \frac{(\$5000)(0.04)}{(1.21665) - 1}$$

$$= \$923.15$$

Present worth of annuity



Present worth of annuity

$$P = A\left[\frac{(1+i)^n - 1}{(i)(1+i)^n}\right]$$

A person wants to borrow as much money as possible today with an annual payment of \$1000 at the end of each year for 5 years. If he is charged 7% interest compounded annually, how much could he borrow?

In this problem $A = \$1000$, $i = 0.07$, $n = 5$, and $P = ?$

$$P = A\left[\frac{(1+i)^n - 1}{(i)(1+i)^n}\right] = (\$1000)\left[\frac{(1.07)^5 - 1}{(0.07)(1.07)^5}\right] = (\$1000)\left[\frac{(0.40255)}{(0.09818)}\right]$$

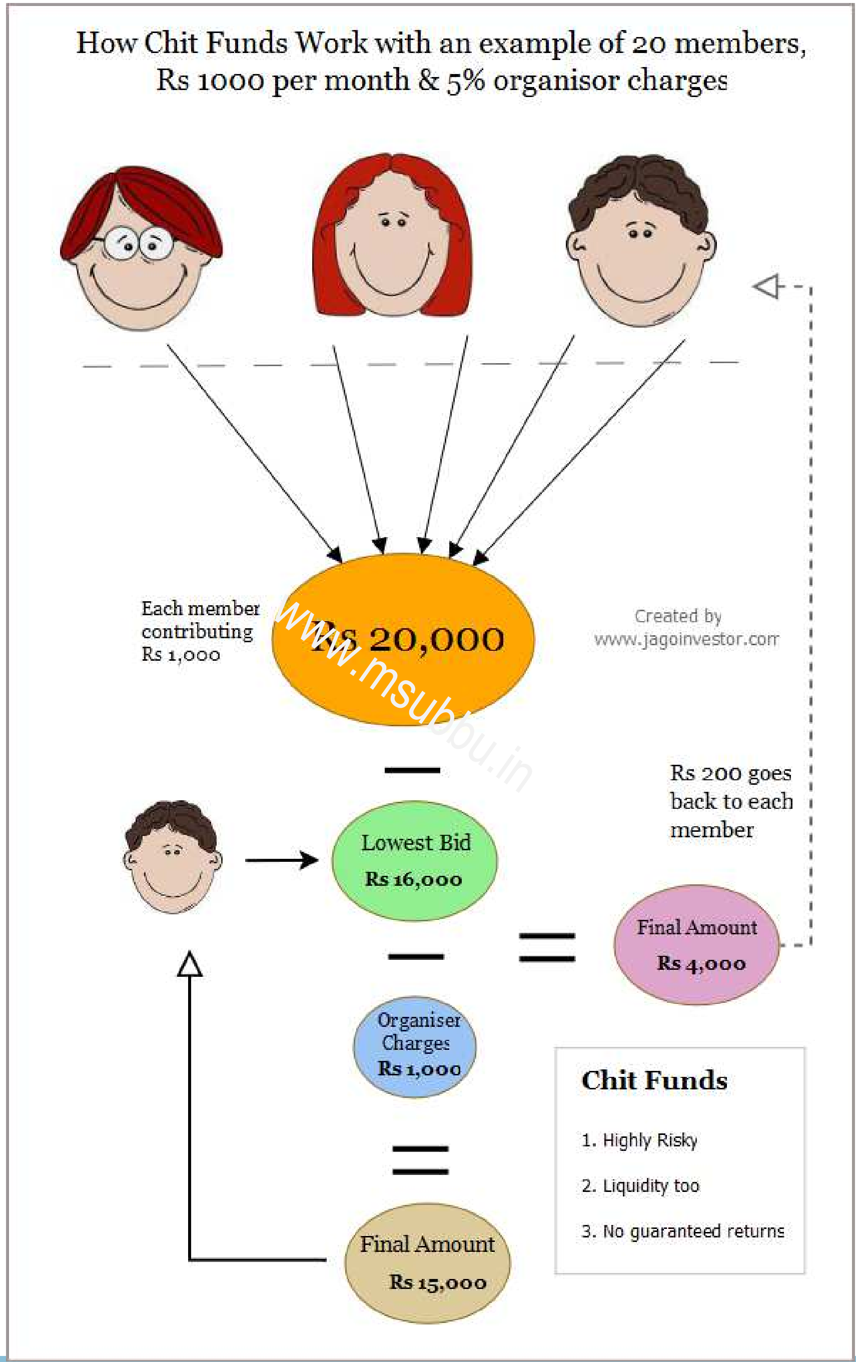
$$= \$4130.25$$

Uniform monthly payment

- Industrial loans may be compounded monthly, quarterly, semiannually, or annually. Personal loans, home mortgages and automobile loans are compounded monthly.

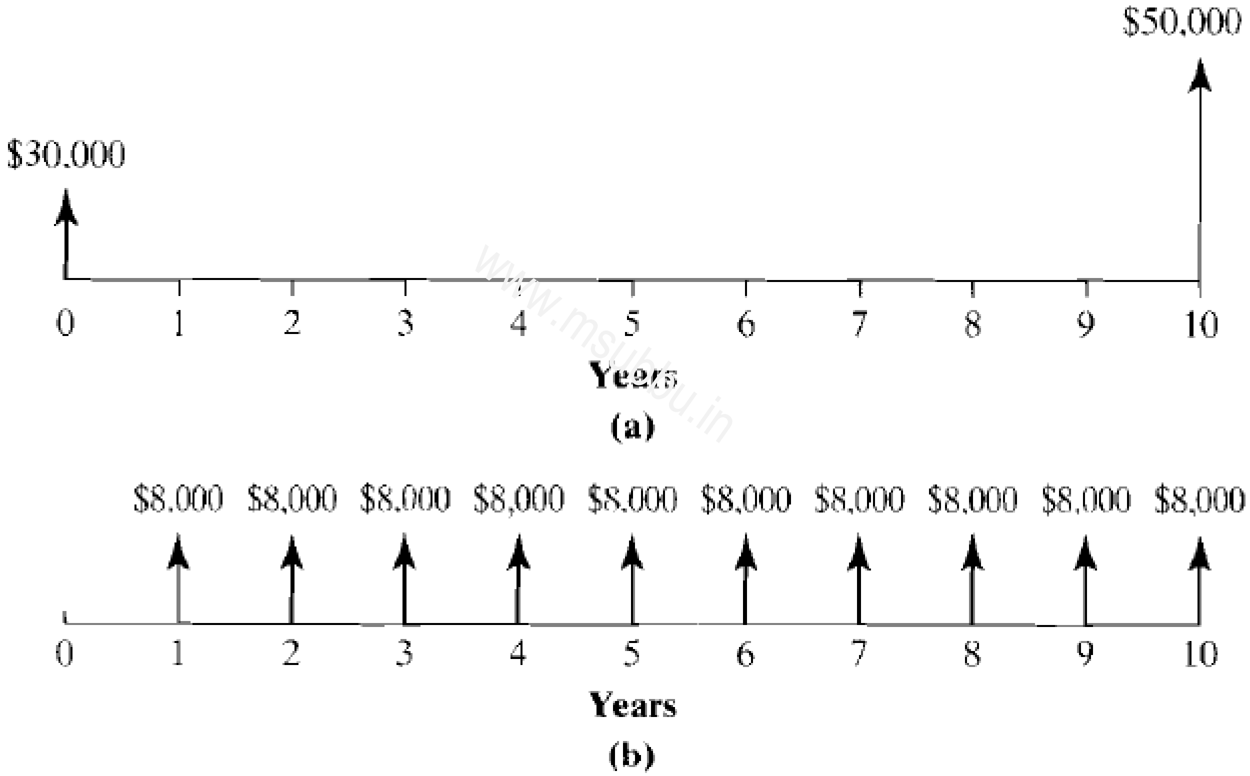
Suppose a person wants to obtain a home equity loan of \$18,500 for remodeling. The interest rate is 8.5% compounded monthly and it is agreed to retire the loan in 5 years. How much will the monthly payments be?

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = 18500\left[\frac{(0.085/12)(1+(0.085/12))^{60}}{(1+(0.085/12))^{60}-1}\right] = \$379.52$$


How Chit Funds Work with an example of 20 members,
Rs 1000 per month & 5% organisor charges
Each member
contributing
Rs 1,000
Rs 20,000
Created by
www.jagoinvestor.com
www.msubbu.in
Rs 200 goes
back to each
member
Lowest Bid
Rs 16,000
Final Amount
Rs 4,000
Organiser
Charges
Rs 1,000
Chit Funds
1. Highly Risky
2. Liquidity too
3. No guaranteed returns
Final Amount
Rs 15,000

Which option would you prefer? (a) Two payments ($30,000 now and $50,000 at the end of 10 years) or (b) 10 equal annual receipts in the amount of $8,000 each

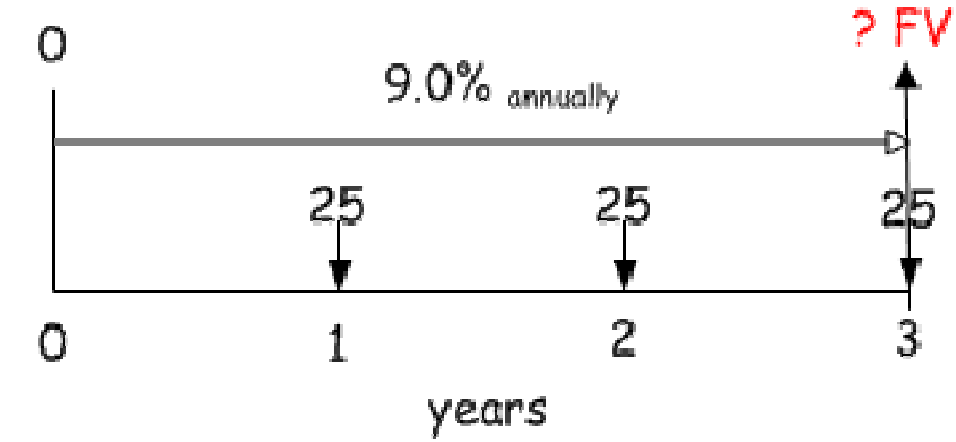
3



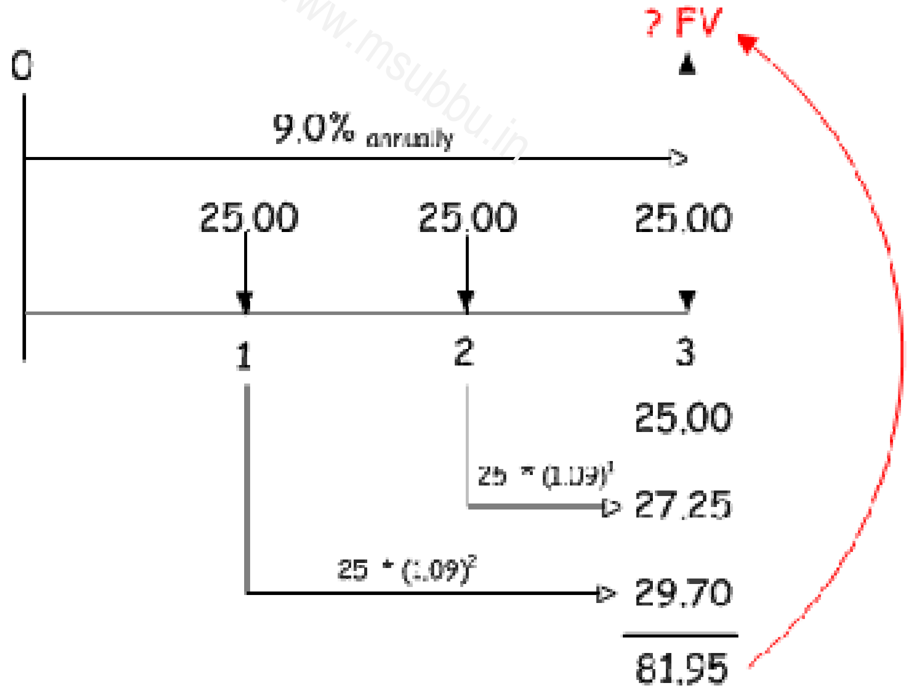
Interest rate is 10% per year

Time Value of Money Calculators

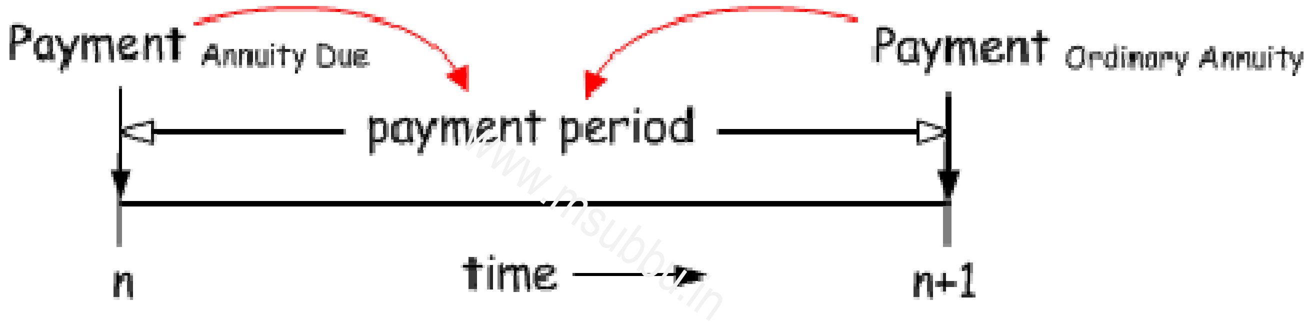
- http://www.calcudora.com/
- http://vindeep.com/Calculators/RDCalc.aspx



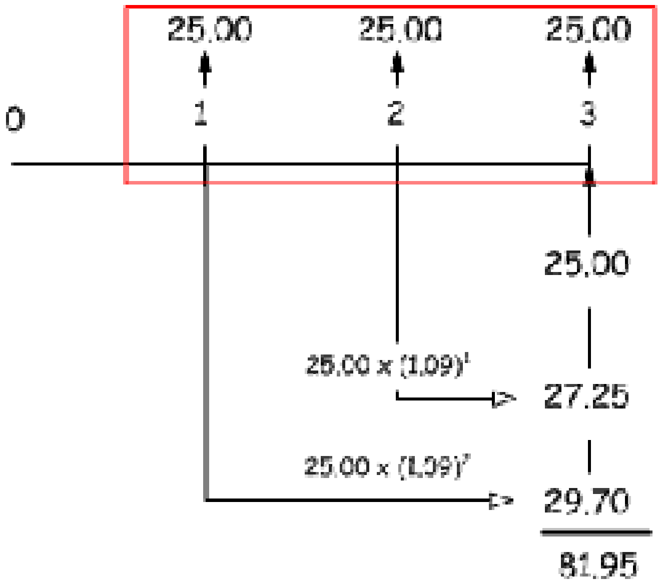
$$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right) = 25.00\left(\frac{(1+0.09)^3 - 1}{0.09}\right) = 81.9525$$

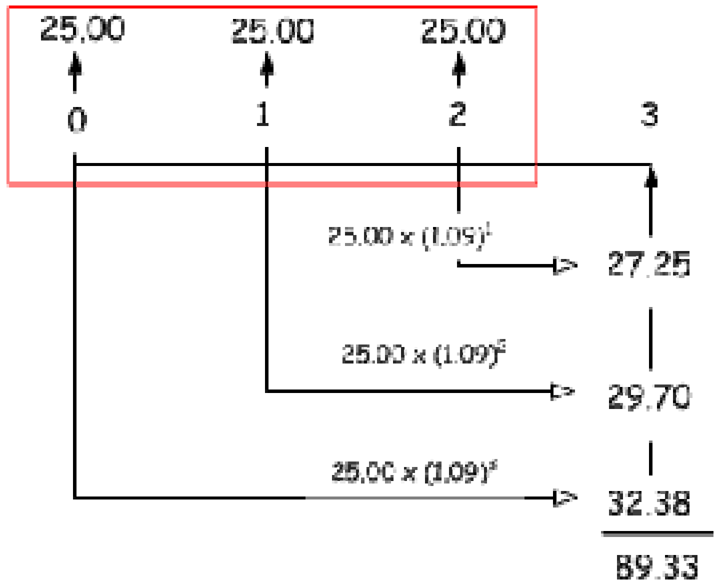


Distinction between an Ordinary Annuity and an Annuity-Due



Payments made under an ordinary annuity occur at the end of the period while payments made under an annuity due occur at the beginning of the period.





$$F_{due} = F_{ordinary}(1+i)$$

http://www.frickcpa.com/tvom/TVOM_Compound.asp